Fatigue of Weldments

June 2nd – 6th, 2014,

Aalto University,

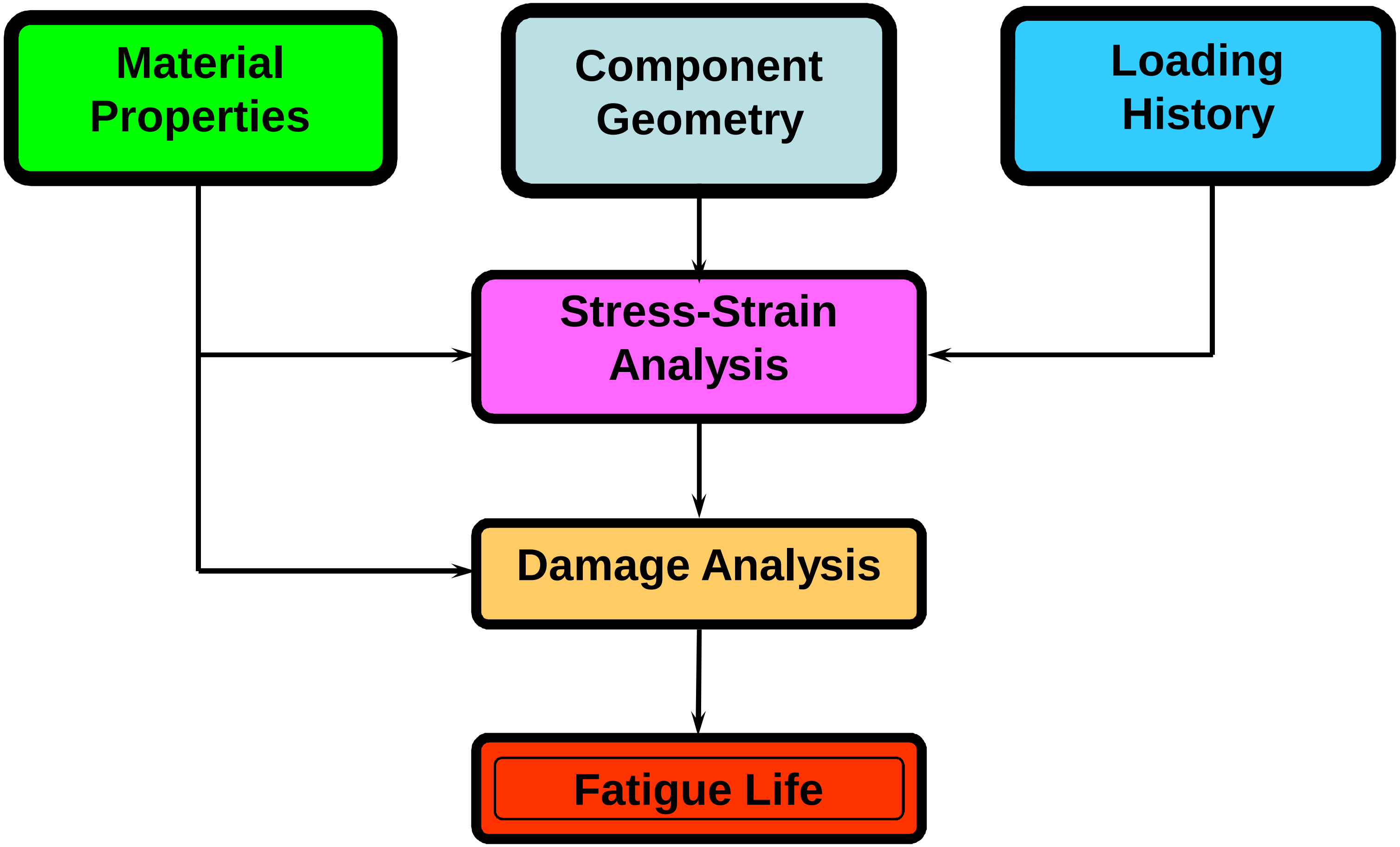
Espoo, Finland

Prof. **Grzegorz Glinka**

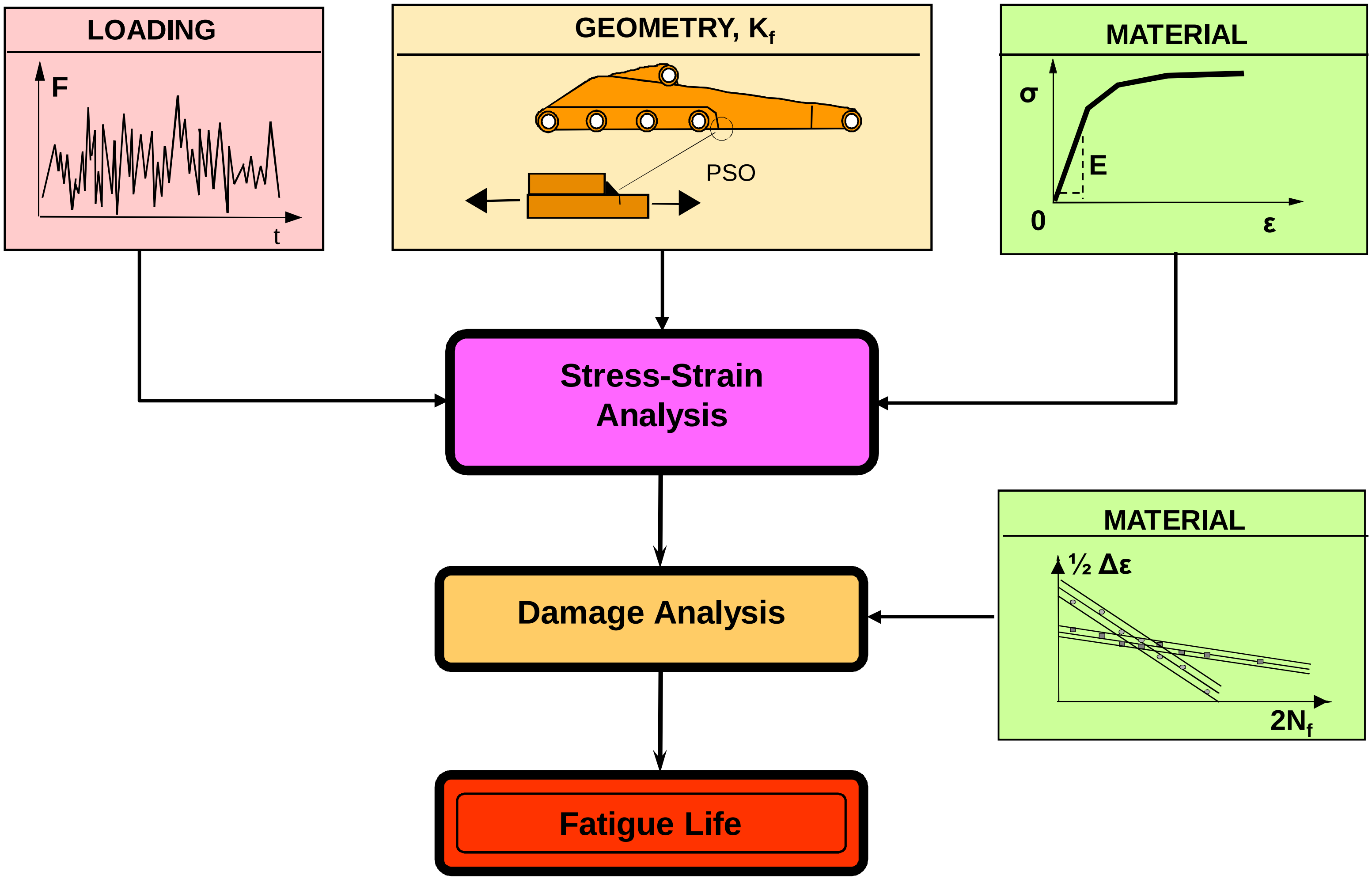
University of Waterloo, Canada

Fatigue Analysis of Weldments by the Local Stress-Strain Method (ε-N)

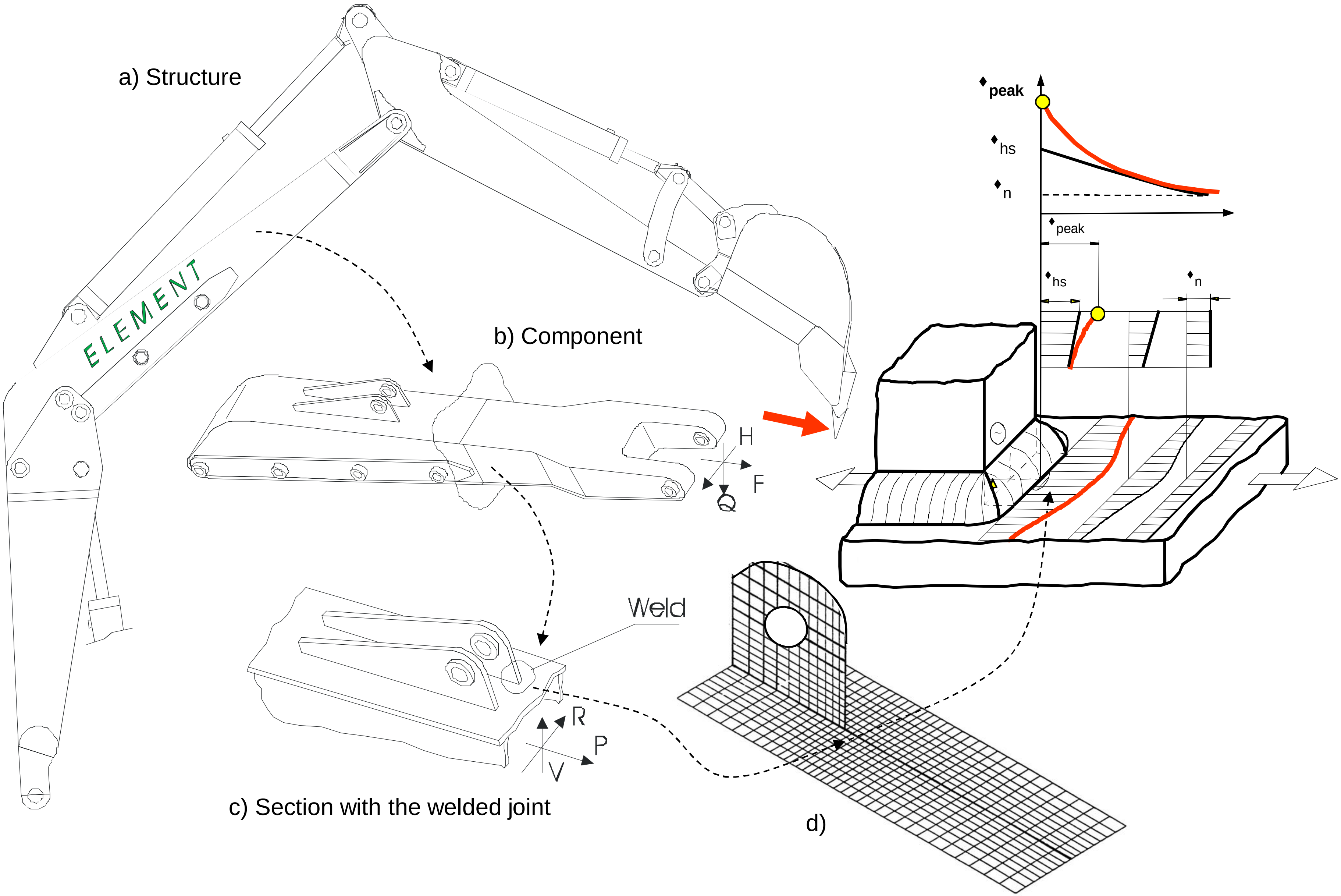
Information Path for Strength and Fatigue Life Analysis



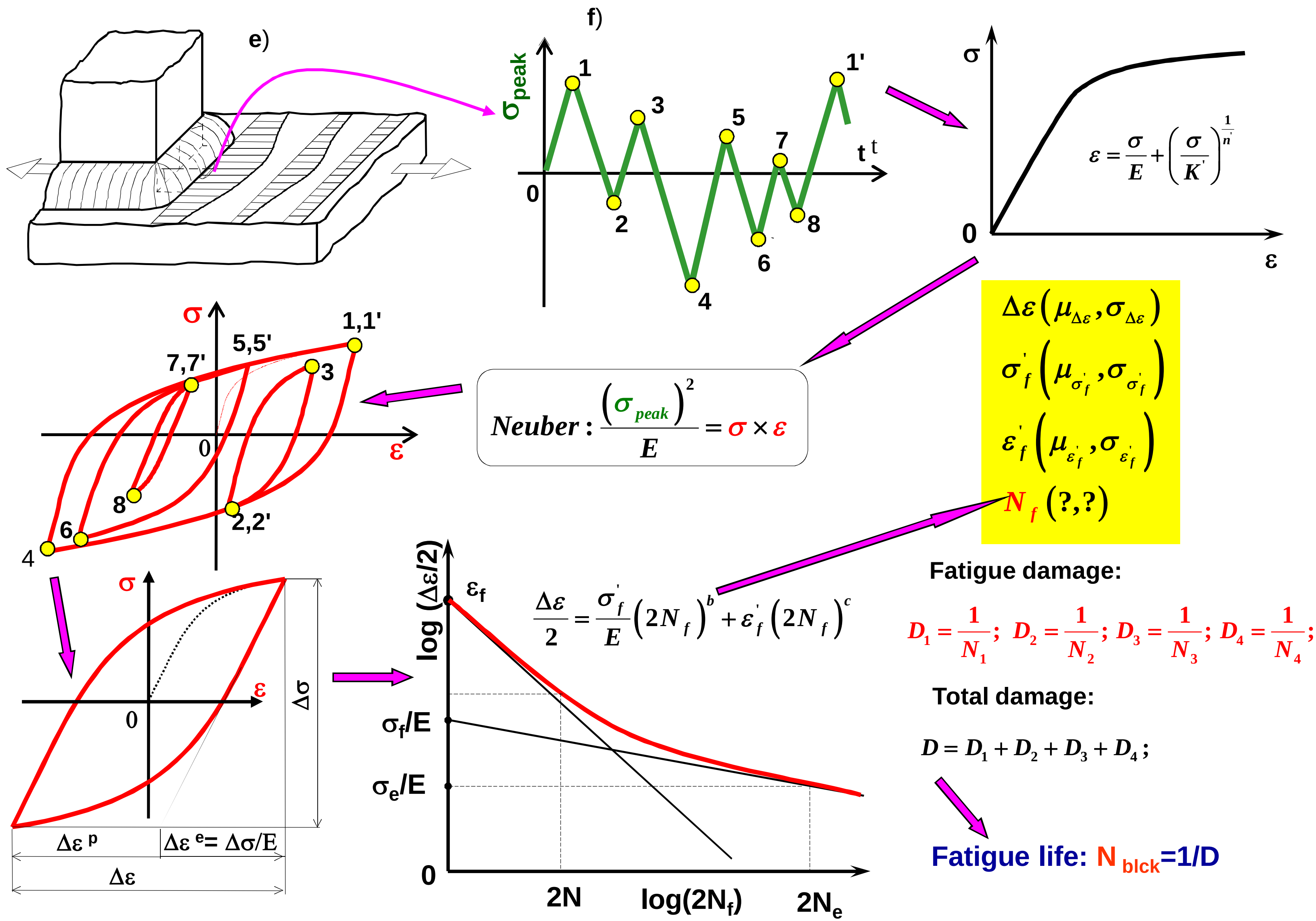
Information path for fatigue life estimation based on the ε-N method



Steps in fatigue life prediction procedure based on the ε - N approach



Steps in fatigue life prediction procedure based on the ε-N approach



The stepwise ε - N procedure for estimating fatigue life (*can be summarised as follows - see the Figure below).*

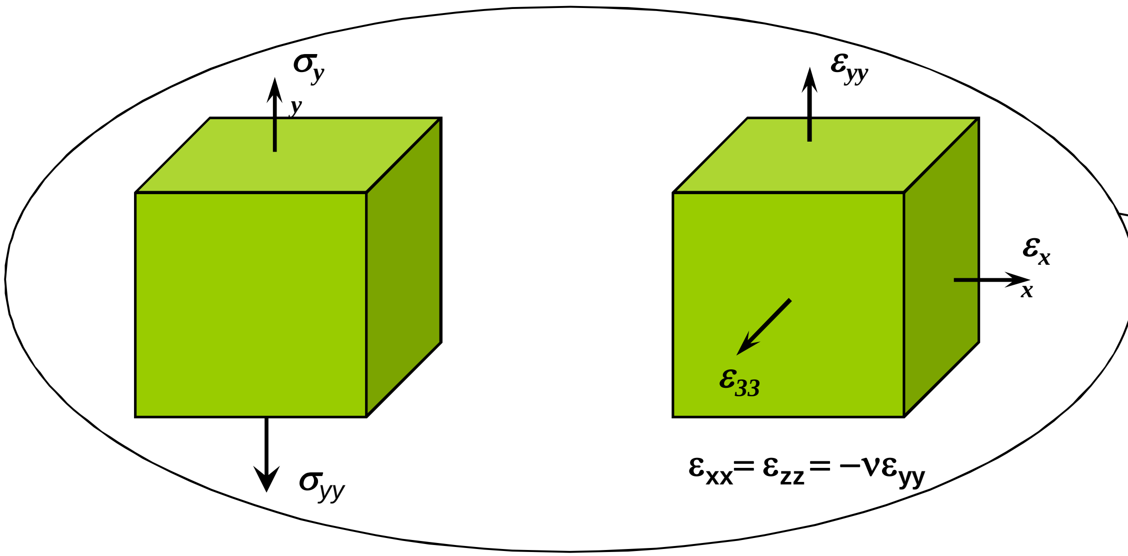
- Analysis of external forces acting on the structure and the component in question (a),
- Analysis of internal loads in chosen cross section of a component (b),
- Selection of critical locations (stress concentration points) in the structure (c),
- Calculation of the elastic local stress, σ_{peak}, at the critical point (usually the notch tip, d)
- Assembling of the local stress history in form of the form of peak and valley sequence (f),
- Determination of the elastic-plastic response at the critical location (h),
- Identification (extraction) of cycles represented by closed stress-strain hysteresis loops (h, i),
- Calculation of fatigue damage (k),
- Fatigue damage summation (Miner- Palmgren hypothesis, l),
- Determination of fatigue life (m) in terms of number of stress history repetitions, N_{blck}, (No. of blocks) or the number of cycles to fatigue crack initiation, N.

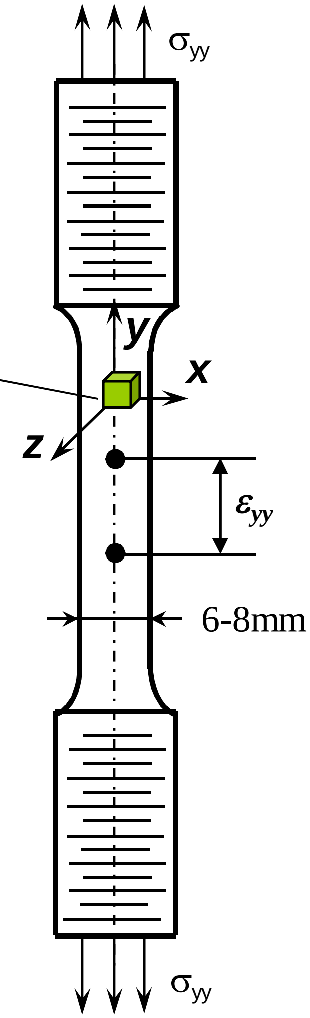
The details concerning many other aspects of that methodology are discussed below.

Material properties used in the strain-life (ε-N) fatigue analysis of weldments

Smooth Laboratory Specimens Used for the Determination of the σ - ε Curve under Monotonic and Cyclic Loading

Stress and strain state in specimens used for determination of material properties

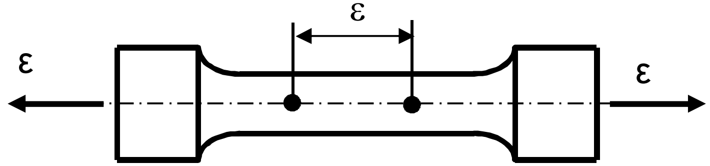


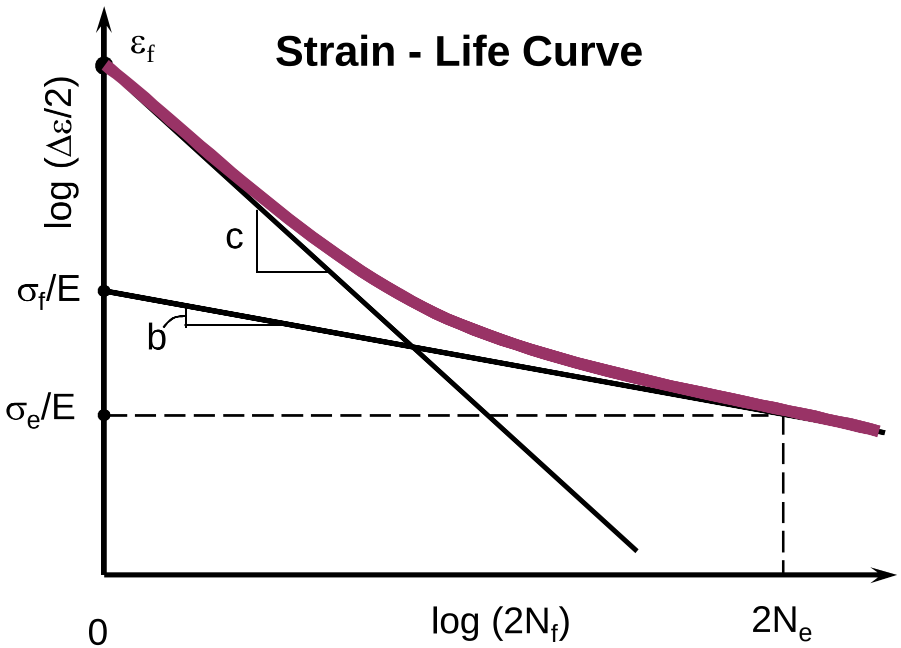


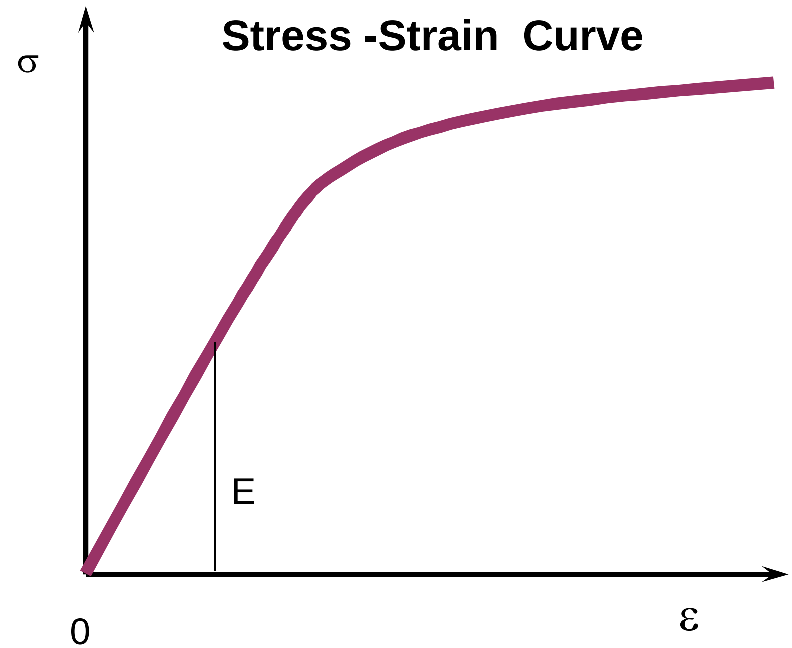
$$\sigma_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \varepsilon_{ij} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}$$

The Strain-life and the Cyclic Stress-Strain Curve Obtained from Smooth Cylindrical Specimens Tested Under Strain Control

(Uni-axial Stress State)



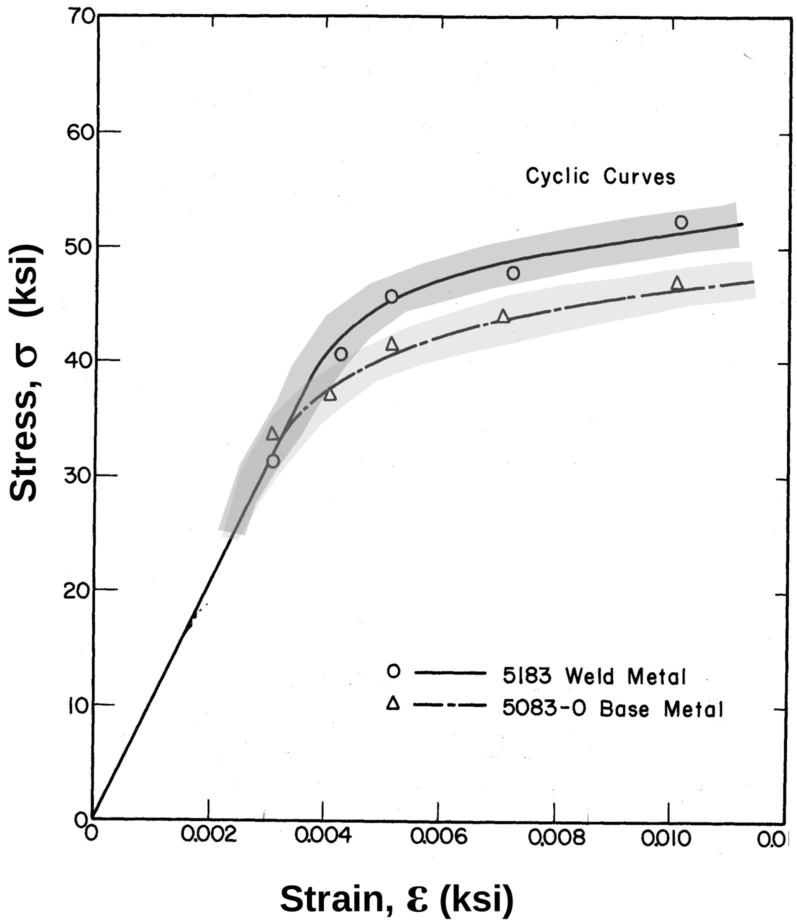


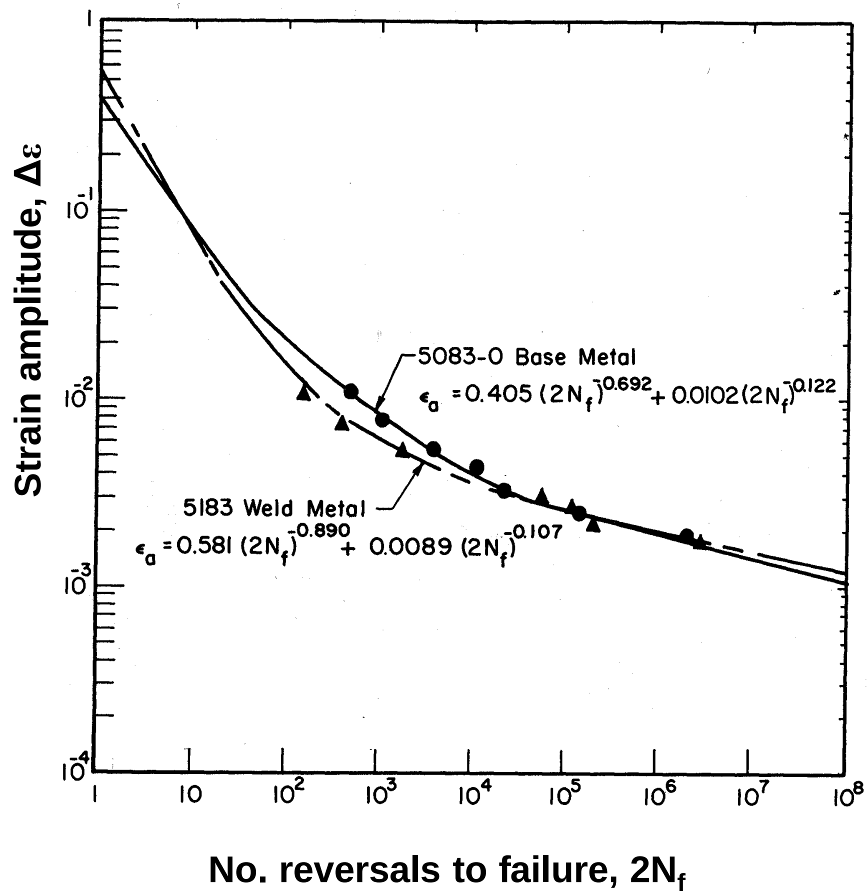


$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c$$

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{\frac{1}{n'}}$$

The effect of the weld and the base material properties on the strain-stress and strain-life properties of welded Aluminum 5183 material





The stress-strain and strain-life data sets for the weld metal and the parent material lie in the same scatter band! Therefore the parent material fatigue properties are used for the analysis of fatigue life of weldments. *(source: J.D Burk and F.V. Lawrence, ref. 40)*

a) Fatigue cracks in weldments initiate most often at the weld toe or the weld root, i.e. in the Heat Affected Zone (HAZ).

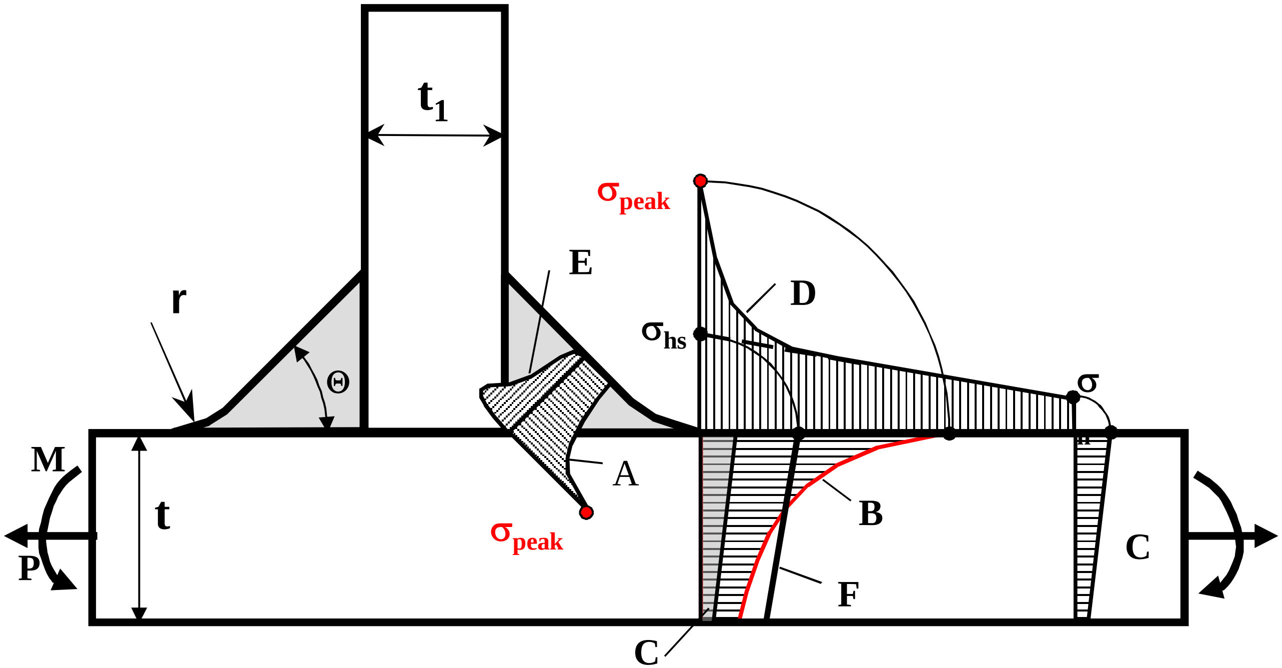
b) Fatigue material properties of the Heat Affected Zone (HAZ) and the Weld Metal (WM) have higher mean values of fatigue strength parameters, than the Base Metal properties, but they are also characterized by wider scatter. The scatter of Base Metal properties often lies within the scatter of the HAZ and WM scatter bands.

c) Therefore the Base Metal cyclic and fatigue properties are most often used for fatigue analyses within the Local Strain (ε-N) method.

Stress parameters used in the strain-life (ε-N) fatigue analysis of weldments

Weldments, like most engineering components, contain stress concentration regions located around weld toes and weld roots. The high local stresses in those locations control the fatigue process of welded components. Therefore the stress peak at the weld root or toe must be determined or accounted for within the procedure aimed at the evaluation of fatigue lives of weldments.

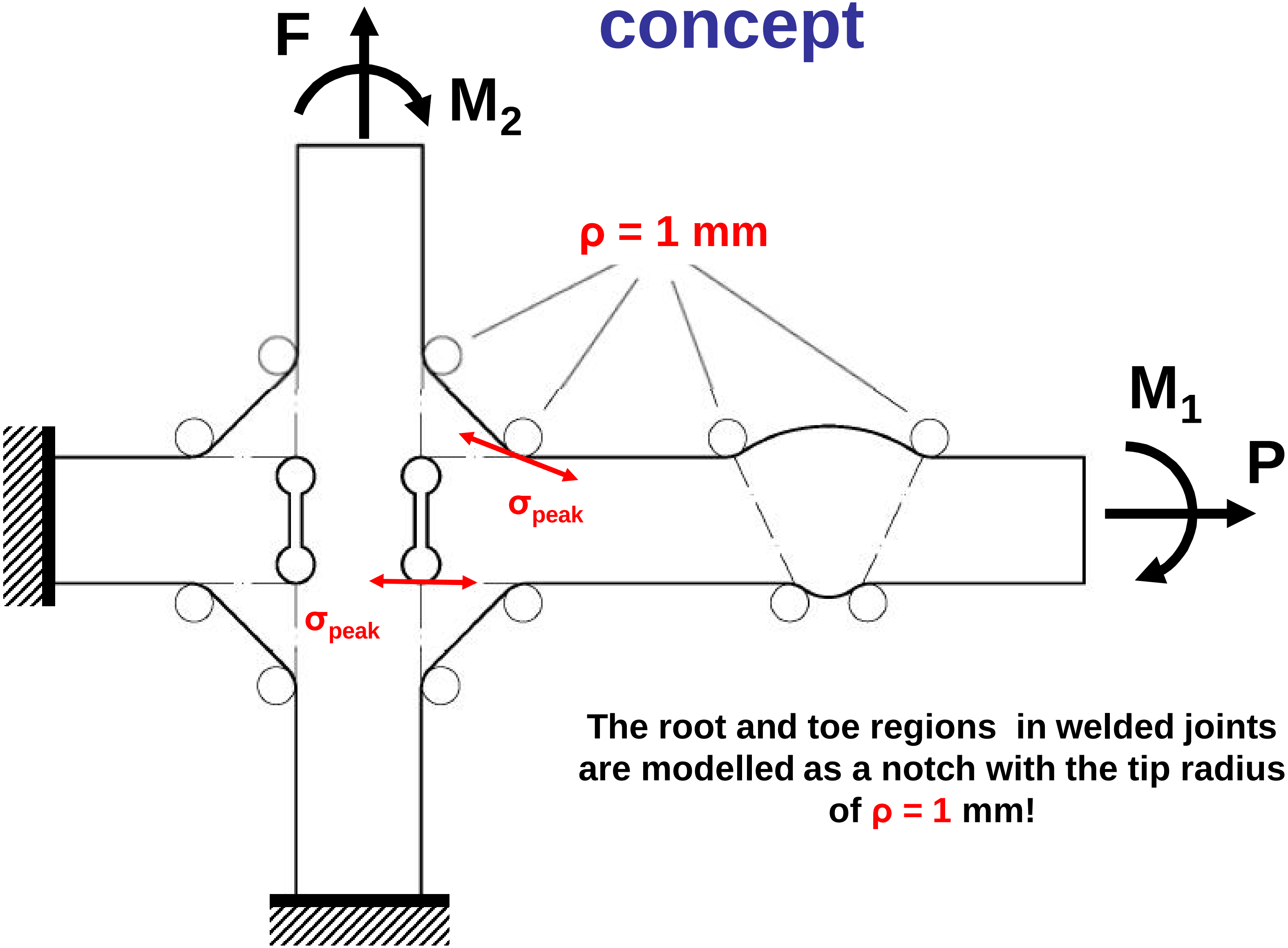
Stress concentration & stress distributions in weldments



Various stress distributions in a T-butt weldment with transverse fillet welds;

- Normal stress distribution in the weld throat plane (**A**),
- Through the thickness normal stress distribution in the weld toe plane (**B**),
- Through the thickness normal stress distribution away from the weld (**C**),
- Normal stress distribution along the surface of the plate (**D**),
- Normal stress distribution along the surface of the weld (**E**),
- Linearized normal stress distribution in the weld toe plane (**F**).

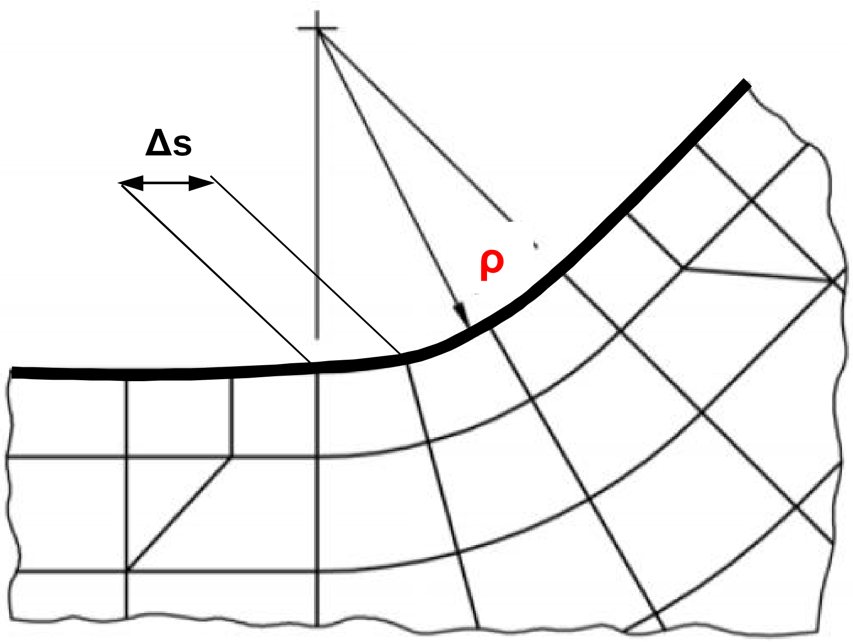
The IIW reference 1-mm toe and root radius concept



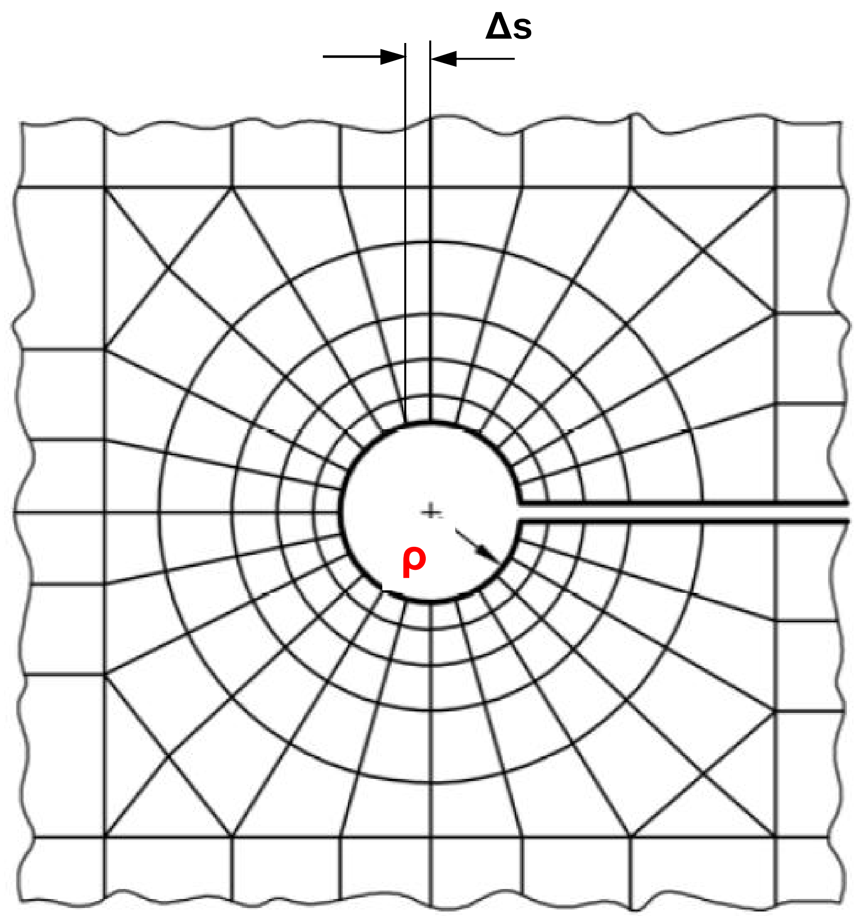
The root and toe regions in welded joints are modelled as a notch with the tip radius of $\rho = 1$ mm!

Recommended FE mesh models for the stress analysis around weld toes and roots having the effective ρ = 1mm tip radii

Recommended element size in the tip region Δs < ρ/4 !

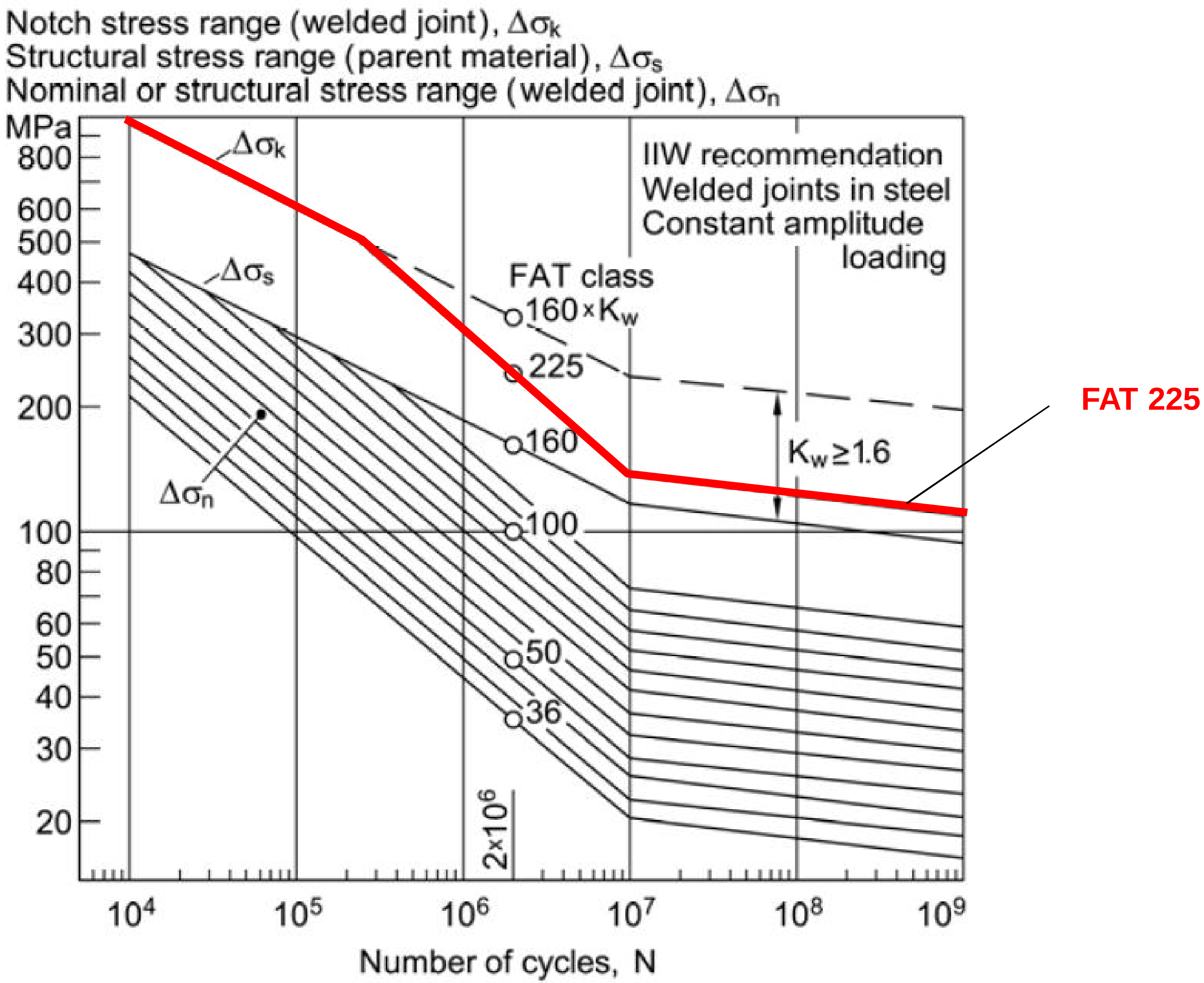


Typical FE mesh for the notch stress analysis around the weld toe region (elements with quadratic shape function)



Typical FE mesh for the notch stress analysis around the weld root region (elements with quadratic shape function)

The recommended IIW fatigue S-N curve associated with the effective ρ=1 mm weld toe and root radius



The Universal GY2 stress analysis method appropriate for any contemporary fatigue analysis method of weldments

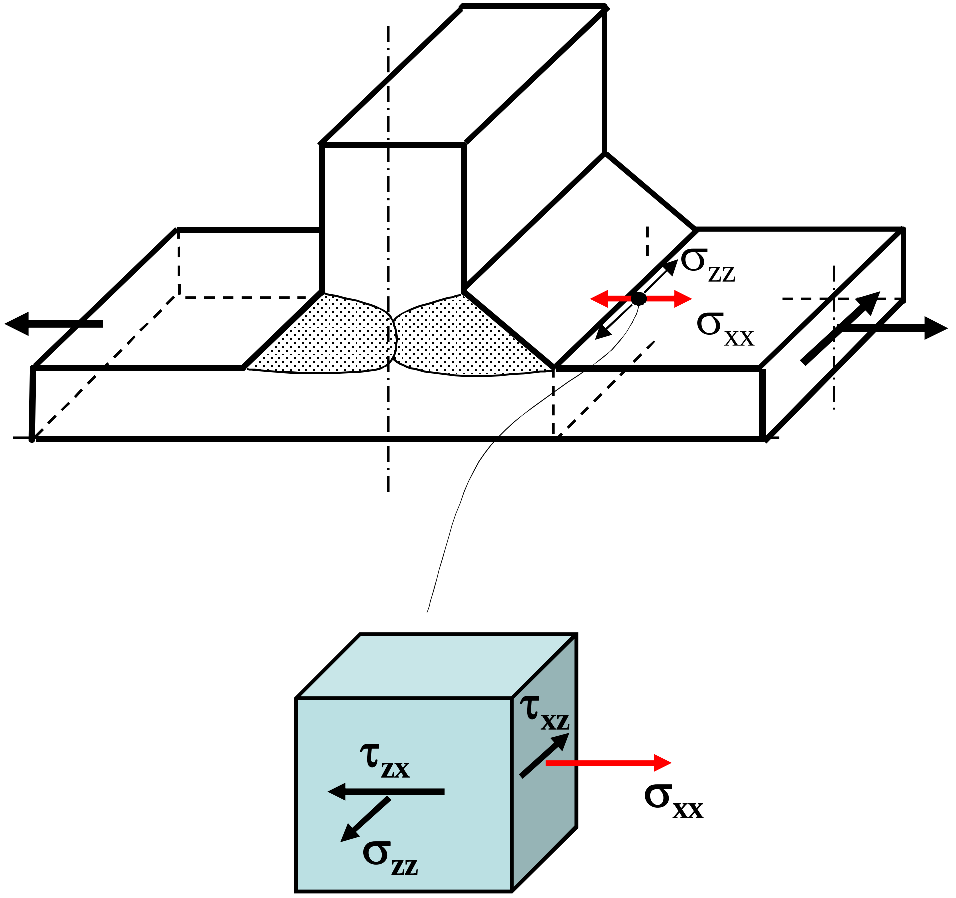
-Nominal stress, S-N

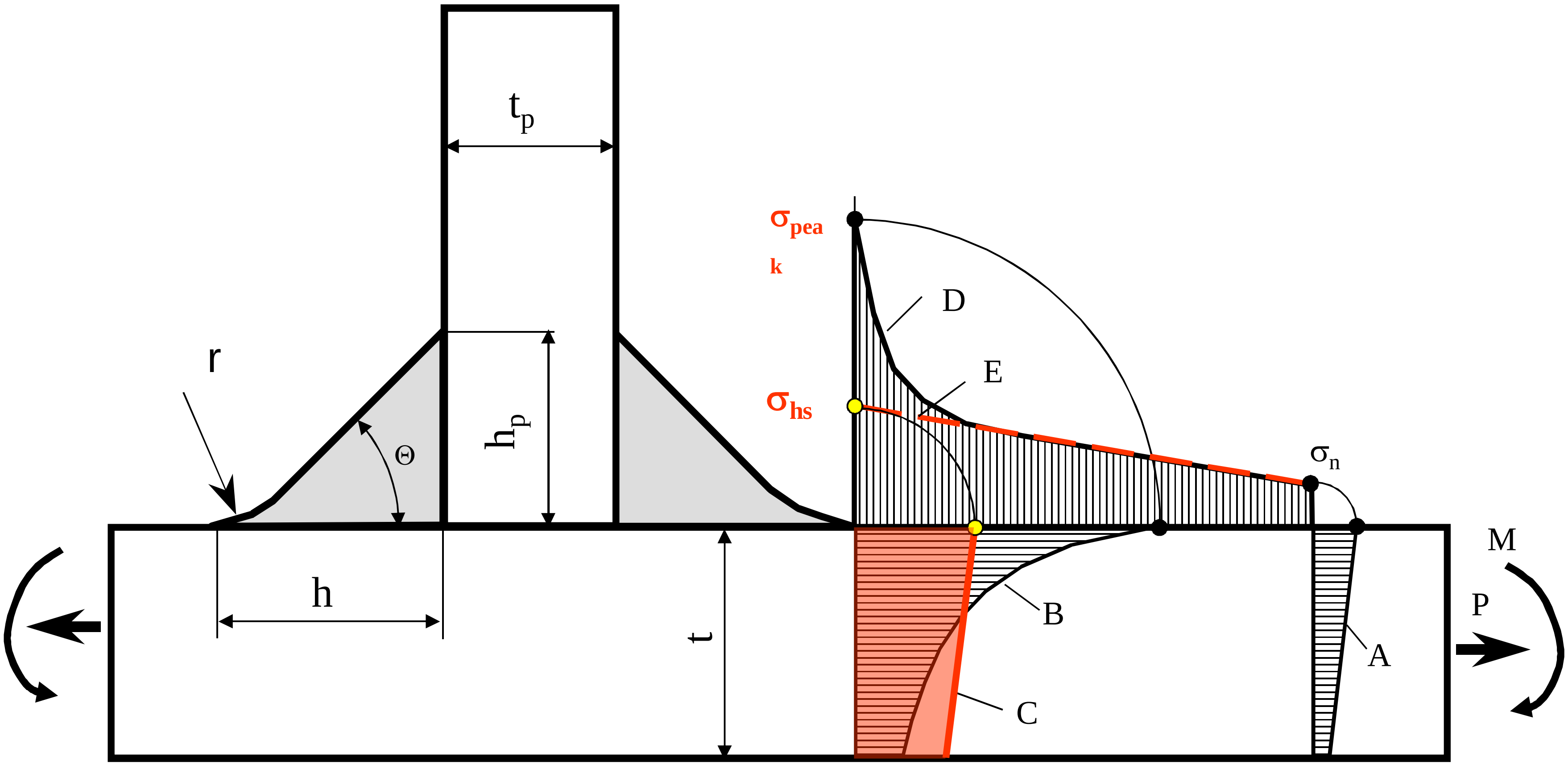
-Local elastic-plastic strain and stress, ε-N

-Fracture mechanics, da/dN-ΔK

The stress state at the weld toe

- **Multiaxial state of stress at weld toe**
- **One shear and two normal stresses**
- **Due to stress concentration, σ_{xx} is the largest component**
  - ***Predominantly responsible for fatigue damage***

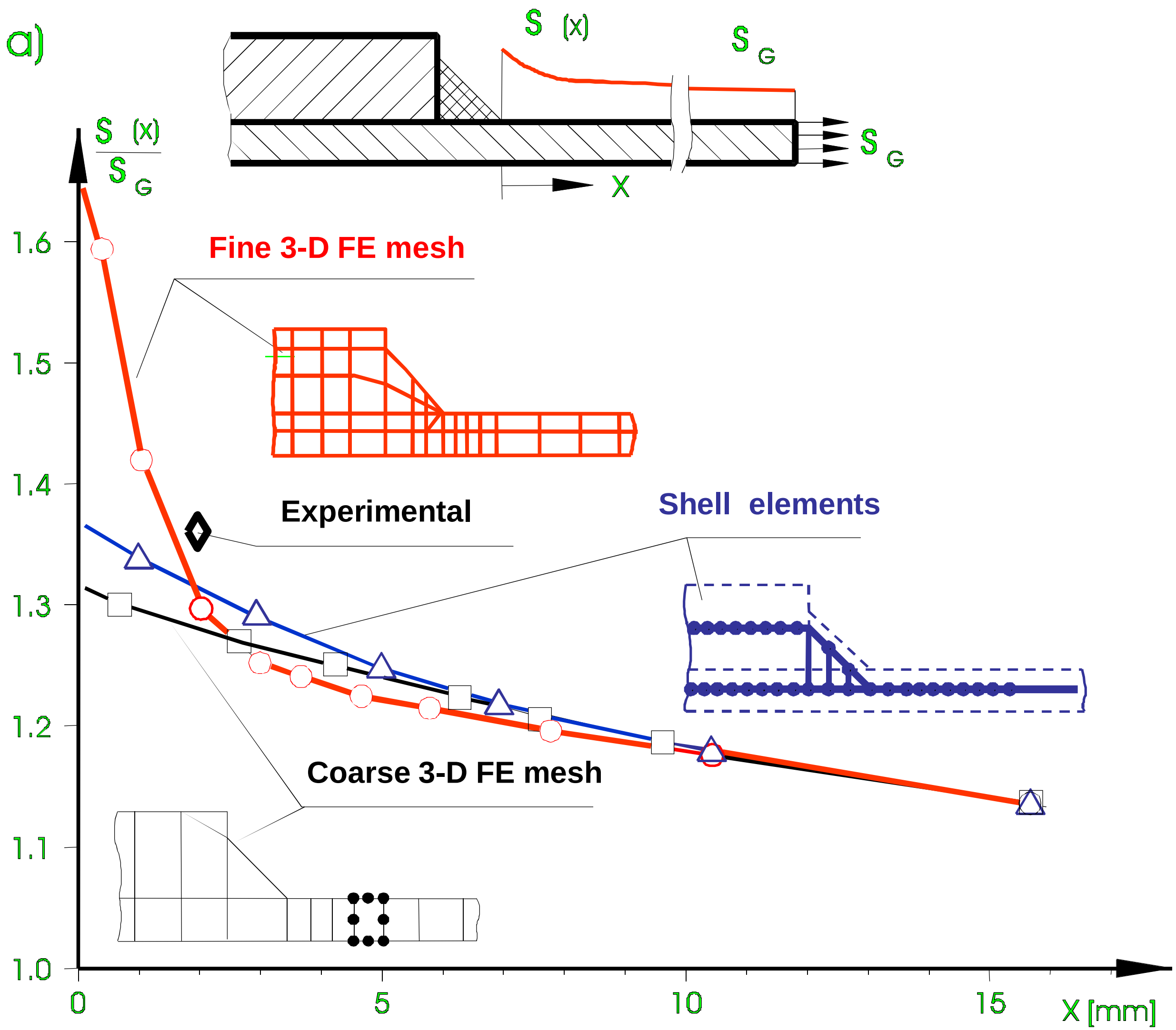




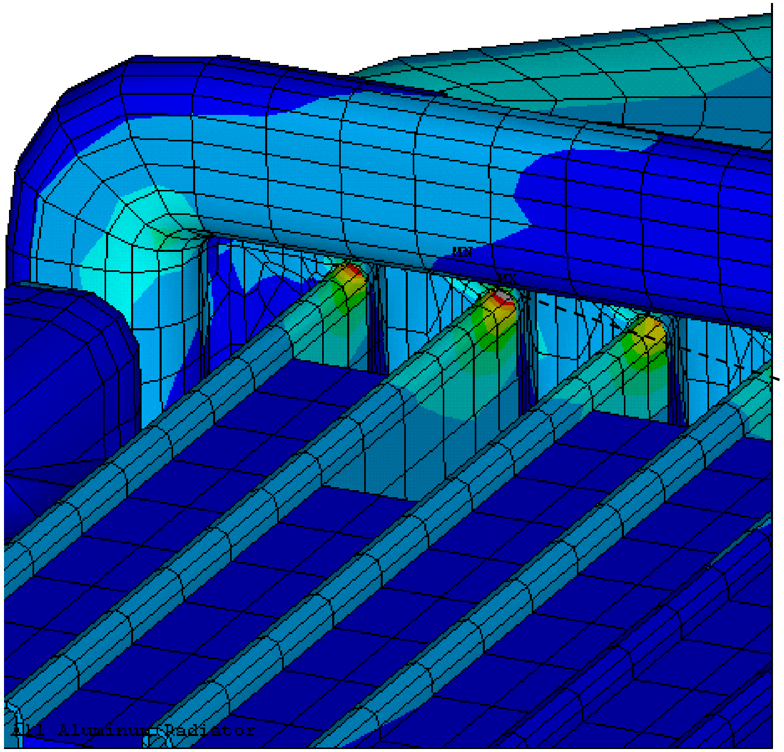
Various stress distributions in a T-butt weldment with transverse fillet welds; A) remote (nominal) through thickness stress, B) the actual through-thickness stress distribution in the weld toe cross section, C) linearized through-thickness stress distribution in the weld toe cross section, D) the actual stress distribution in the plate surface, E) extrapolated (linearly) stress distribution in the plate surface

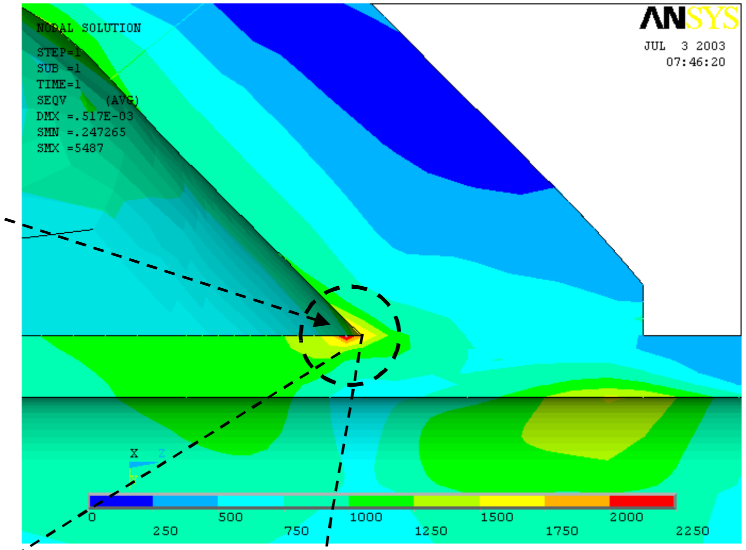
Stress magnitudes and distributions obtained from various FE models:

What stress is the right one for fatigue analyses?

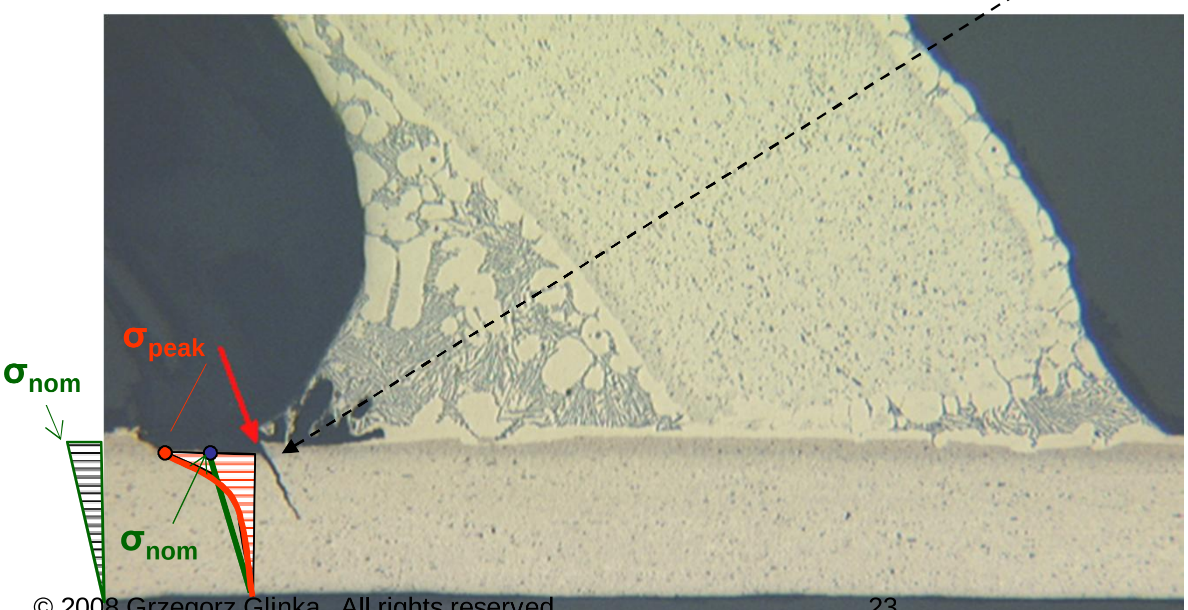


Wrong Finite Element Modeling and wrong resulting stress data!





$\sigma_{peak} \rightarrow \infty$!!

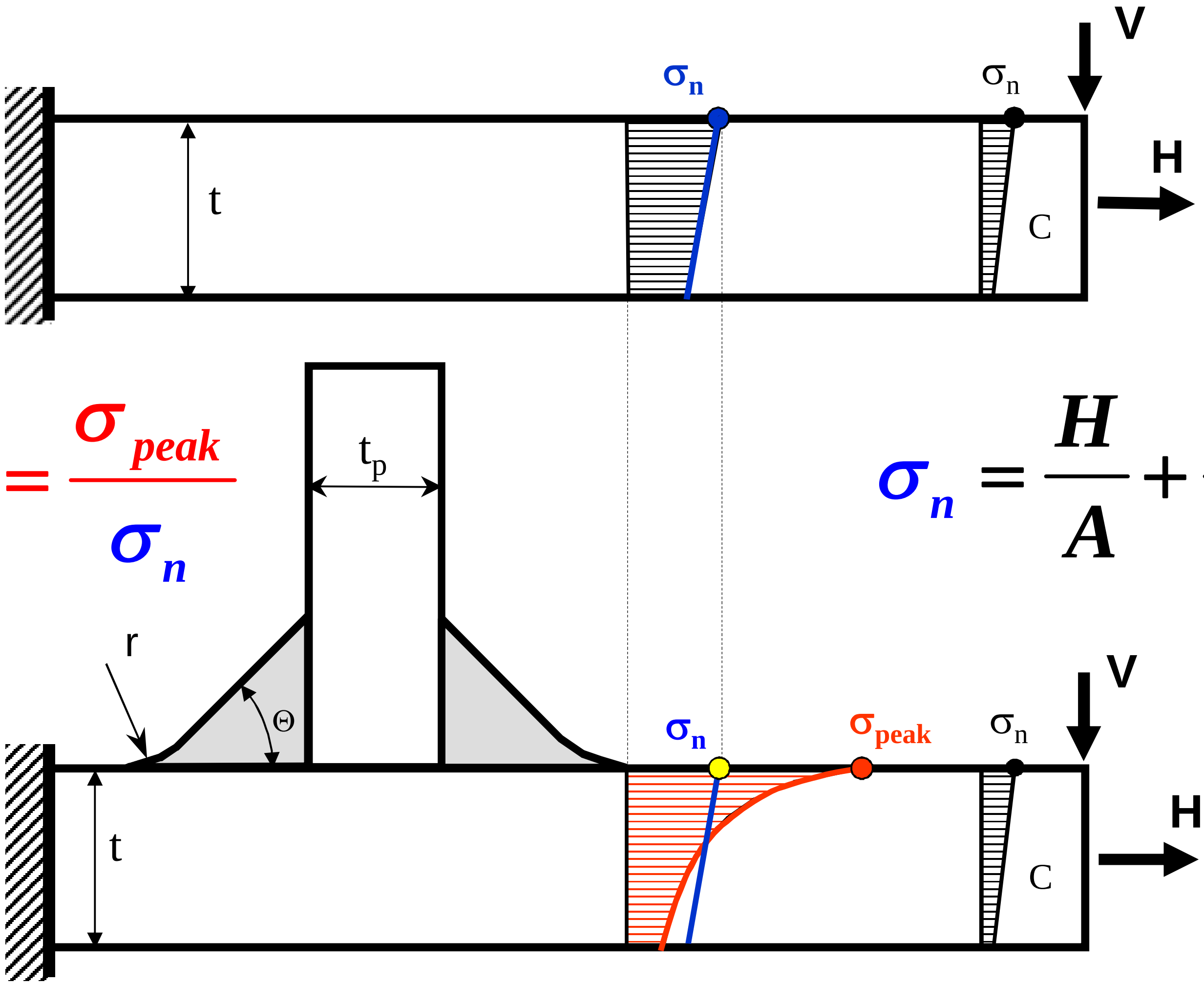


FEM → σ_{peak}

Strain gauge → σ_{nom}

What stress for fatigue life estimations?

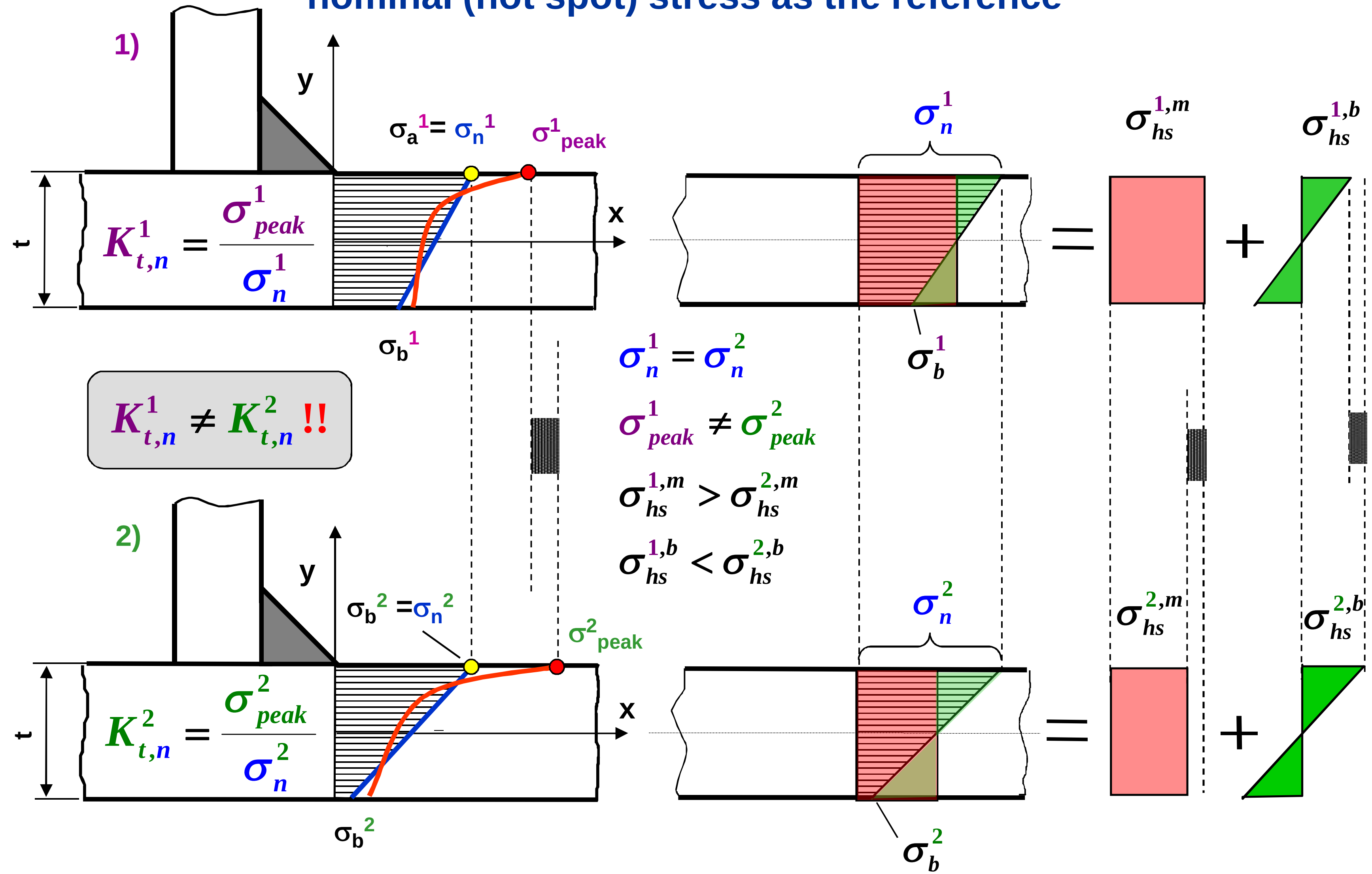
The meaning of the nominal (reference) stress and the stress concentration factor



$$K_{t,n} = \frac{\sigma_{peak}}{\sigma_n}$$

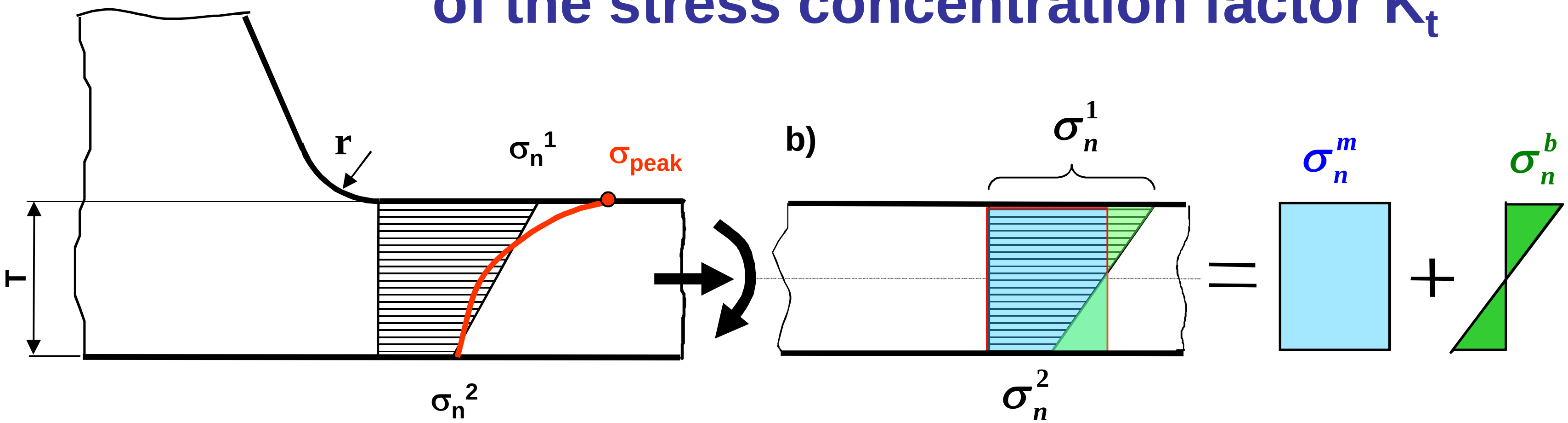
$$\sigma_n = \frac{H}{A} + \frac{M \cdot t}{2I}$$

Non-uniqueness of the stress concentration factor $K_{t,n}$ based on the nominal (hot spot) stress as the reference



The stress concentration $K_{t,n}$ depends on the membrane to bending stress ratio, $\sigma_{hs}^m / \sigma_{hs}^b$!!

Inconsistency concerning the definition of the stress concentration factor K_t



$$\sigma_n^m = \frac{\sigma_n^1 + \sigma_n^2}{2}$$

$$\sigma_n^b = \frac{\sigma_n^1 - \sigma_n^2}{2}$$

$$\sigma_n = \sigma_{hs} = \sigma_n^m + \sigma_n^b = \sigma_n^1$$

$$K_t^{\sigma_n^m} = \frac{\sigma_{peak}}{\sigma_n^m}$$

$$K_t^{\sigma_n^b} = \frac{\sigma_{peak}}{\sigma_n^b}$$

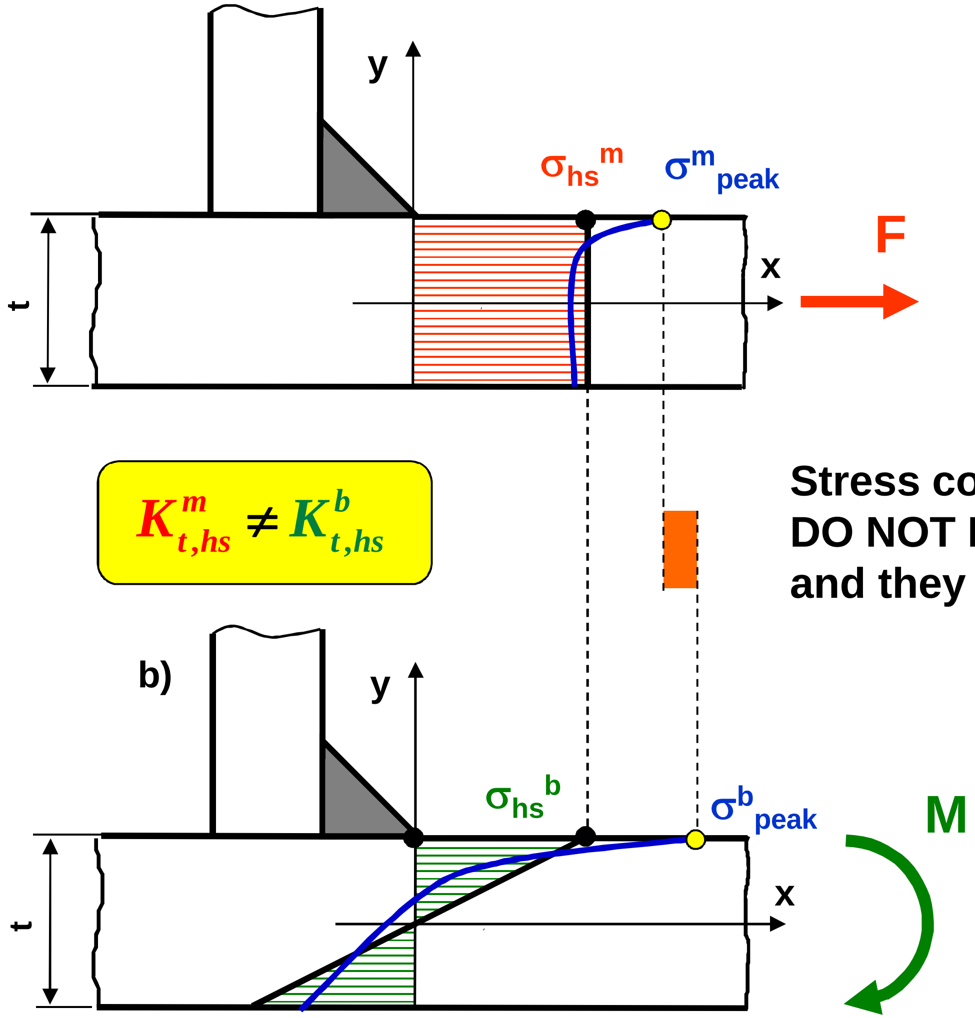
$$K_t = \frac{\sigma_{peak}}{\sigma_n} = \frac{\sigma_{peak}}{\sigma_n^m + \sigma_n^b}$$

The stress concentration factors, $K_t^{\sigma_n^m}$, $K_t^{\sigma_n^b}$ and K_t are not constant and not the same!

They depend on the geometry and on the stress ratio: σ^m_n / σ^b_n!

a) A body with an angular notch subjected to multiple loading modes and resulting through-the-thickness stress distribution, b) decomposition of the nominal (linear) stress distribution in the notch cross section into the membrane and bending contribution

Universal stress concentration factor $K^{m}_{t.hs}$ and $K^{b}_{t.hs}$



$$K^{m}_{t,hs} \neq K^{b}_{t,hs}$$

a) Pure axial load

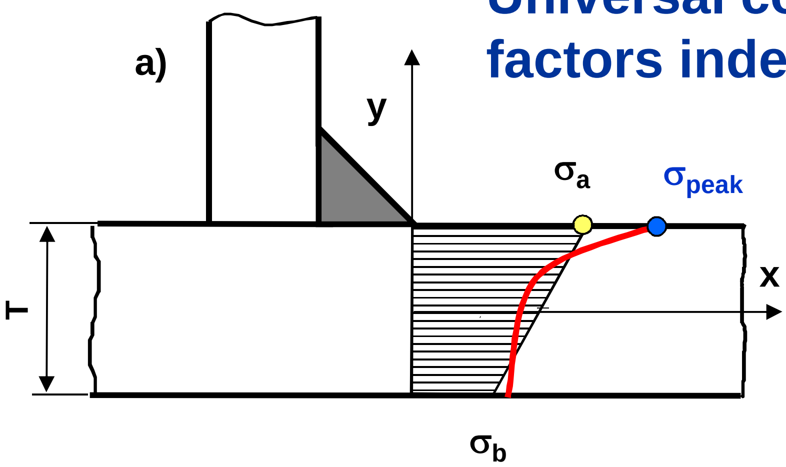
$$K^{m}_{t,hs} = \frac{\sigma^{m}_{peak}}{\sigma^{m}_{hs}}$$

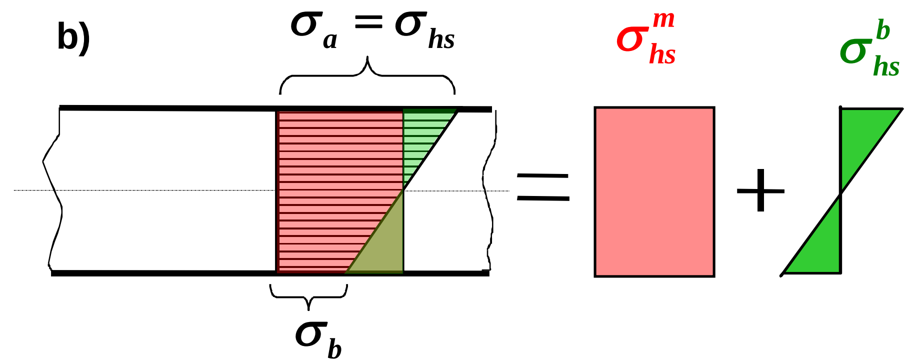
Stress concentration factors $K^{m}_{t,hs}$ and $K^{b}_{t,hs}$ DO NOT DEPEND on the stress ratio $\sigma^{m}_{hs}/\sigma^{b}_{hs}$ and they are constant for given geometry!!

b) Pure bending load

$$K^{b}_{t,hs} = \frac{\sigma^{b}_{peak}}{\sigma^{b}_{hs}}$$

Universal combination of stress concentration factors independent of the loading mode





c)

$$\sigma^m_{hs} = \frac{\sigma_a + \sigma_b}{2}$$

$$\sigma^b_{hs} = \frac{\sigma_a - \sigma_b}{2}$$

$$\sigma_a = \sigma^m_{hs} + \sigma^b_{hs} = \sigma_{hs}$$

$$\sigma_b = \sigma^m_{hs} - \sigma^b_{hs}$$

d)

$$\sigma_{peak} = K^m_{t,hs}\sigma^m_{hs} + K^b_{t,hs}\sigma^b_{hs}$$

The stress concentration factors $K^m_{t,hs}$ and $K^b_{t,hs}$ depend only on the geometry and they can be used for any stress ratio $\sigma^m_{hs} / \sigma^b_{hs}$!!

a) T-butt weldment and resulting through-the-thickness stress distribution, b) decomposition of the nominal (linear) stress distribution in the weld toe plate cross section, c) the hot spot stress as a sum of the hot spot membrane and bending stress, d) the actual peak stress as a sum of the stress concentration on the hot spot membrane and bending stress

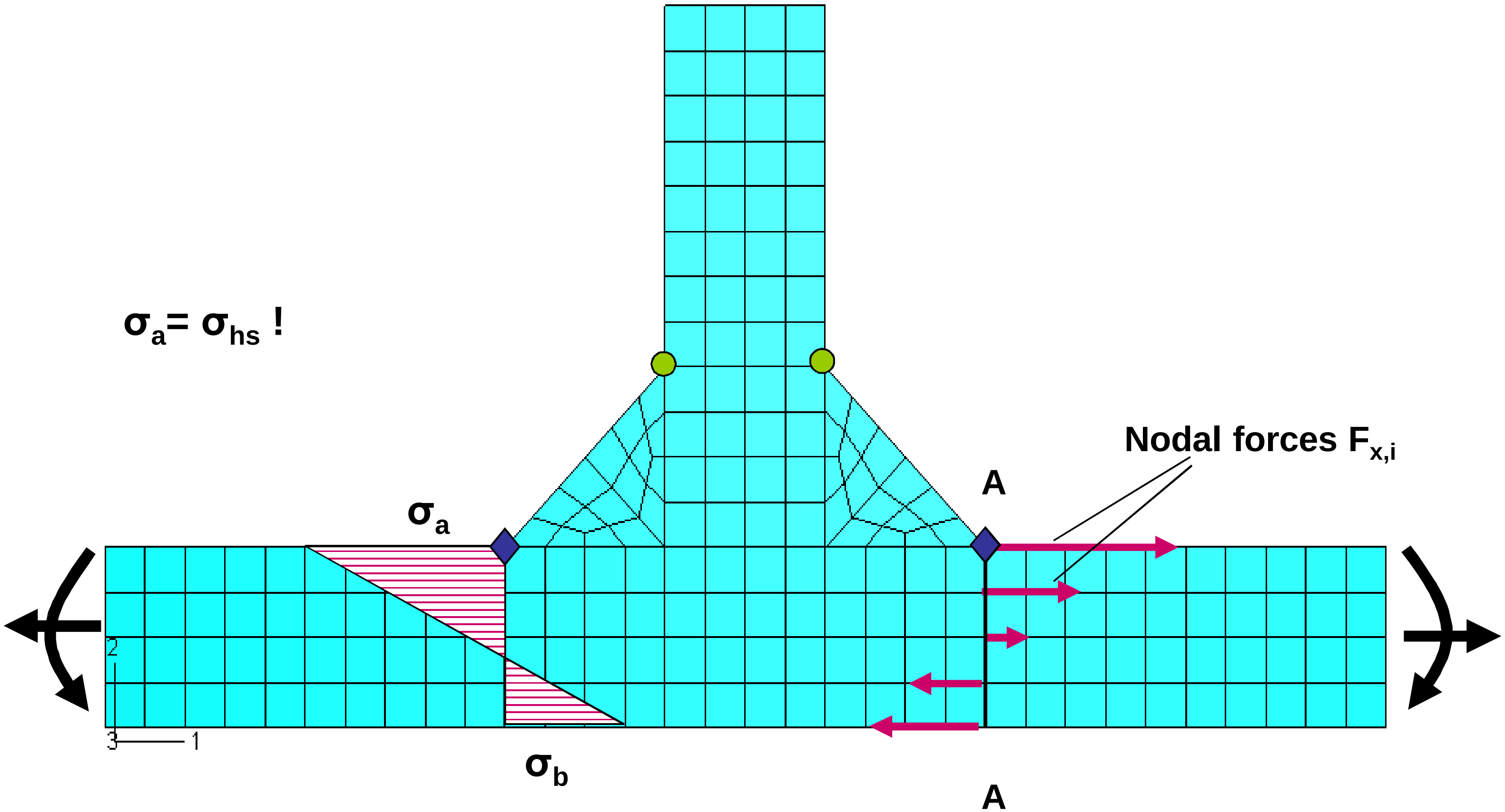
The "hot spot" and the weld toe peak stresses

The advantage of using expression

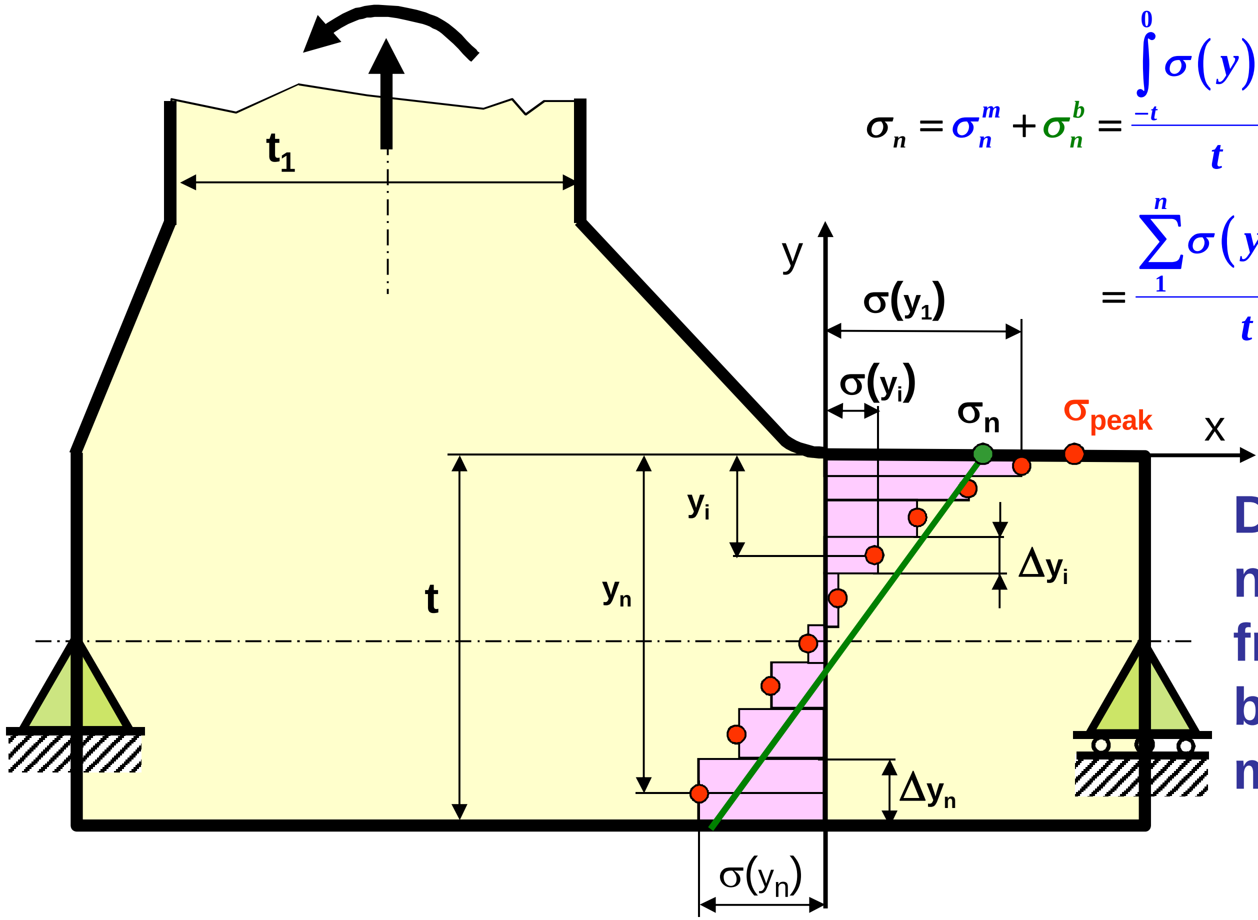
$$\sigma_{peak} = K_{t,hs}^{m} \cdot \sigma_{hs}^{m} + K_{t,hs}^{b} \cdot \sigma_{hs}^{b}$$

lies in the fact that the membrane stress $\sigma_{hs}{}^{m}$ and the bending stress $\sigma_{hs}{}^{b}$ can be determined by simple decomposition of the linearized through-thickness stress field, ***σ(x=0,y)*,** which can be ***directly obtained from the coarse mesh 3-D or shell Finite Element (GY2) analysis***. Thus the equation above provides the link between the FE stress analysis data, $\sigma_{hs}{}^{m}$ and $\sigma_{hs}{}^{b}$, and the peak stress, σ_{peak}, at the weld toe, necessary for the fatigue analysis

Coarse 3D FE mesh model of a welded T-joint



The linearized stress field (σ_a, σ_b) is determined from the distribution of nodal forces $F_{x,i}$! ***(method of D. Pingsha, Batelle Columbus)***



$$\sigma_n = \sigma_n^m + \sigma_n^b = \frac{\int_{-t}^{0} \sigma(y)\,dy}{t} + \frac{6\cdot\int_{-t}^{0} \sigma(y)\,y\,dy}{t^2}$$

$$= \frac{\sum_{1}^{n} \sigma(y)\cdot \Delta y_i}{t} + \frac{6\cdot \sum_{1}^{n} \sigma(y)\cdot y_i \cdot \Delta y_i}{t^2}$$

Determination of nominal stresses from discrete FE data by the linearization method

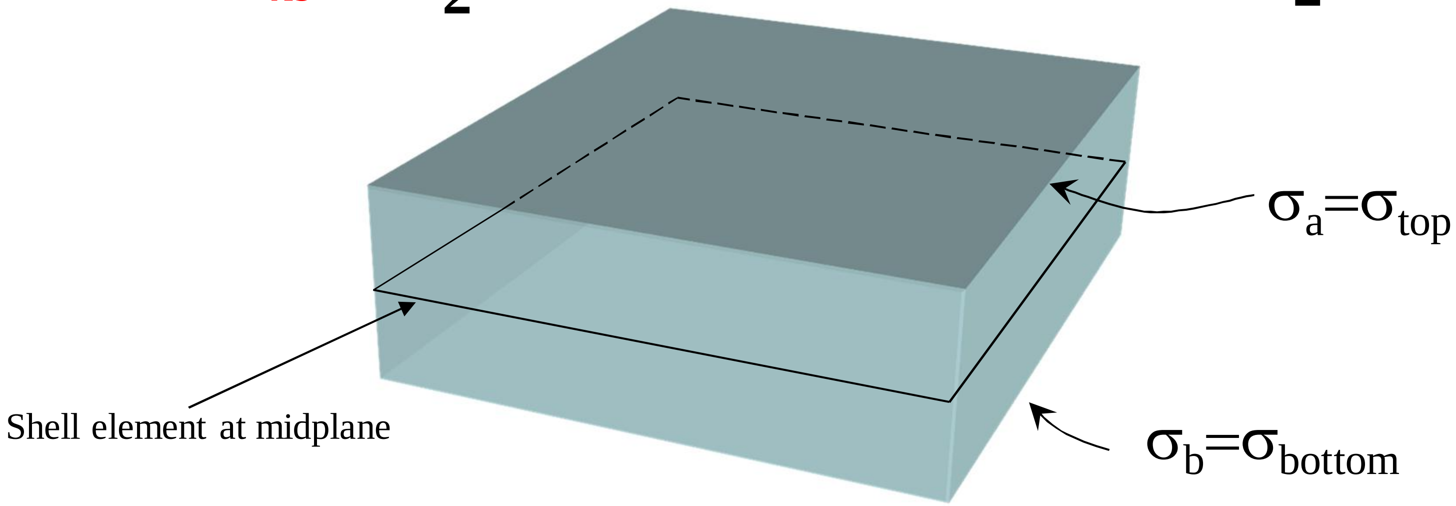
$$\sigma_n^m = \frac{P}{1\cdot t} = \frac{\int_{-t}^{0} \sigma(y)\cdot 1\cdot dy}{1\cdot t} = \frac{\int_{-t}^{0} \sigma(y)\,dy}{t} = \frac{\sum_{1}^{n} \sigma(y)\cdot \Delta y_i}{t};$$

$$\sigma_n^b = \frac{c\cdot M}{I} = \frac{\frac{t}{2}\cdot \int_{-t}^{0} \sigma(y)\cdot 1\cdot y\cdot dy}{\frac{1\cdot t^3}{12}} = \frac{6\int_{-t}^{0} \sigma(y)\cdot y\cdot dy}{t^2} = \frac{6\cdot \sum_{1}^{n} \sigma(y)\cdot y_i\cdot \Delta y_i}{t^2};$$

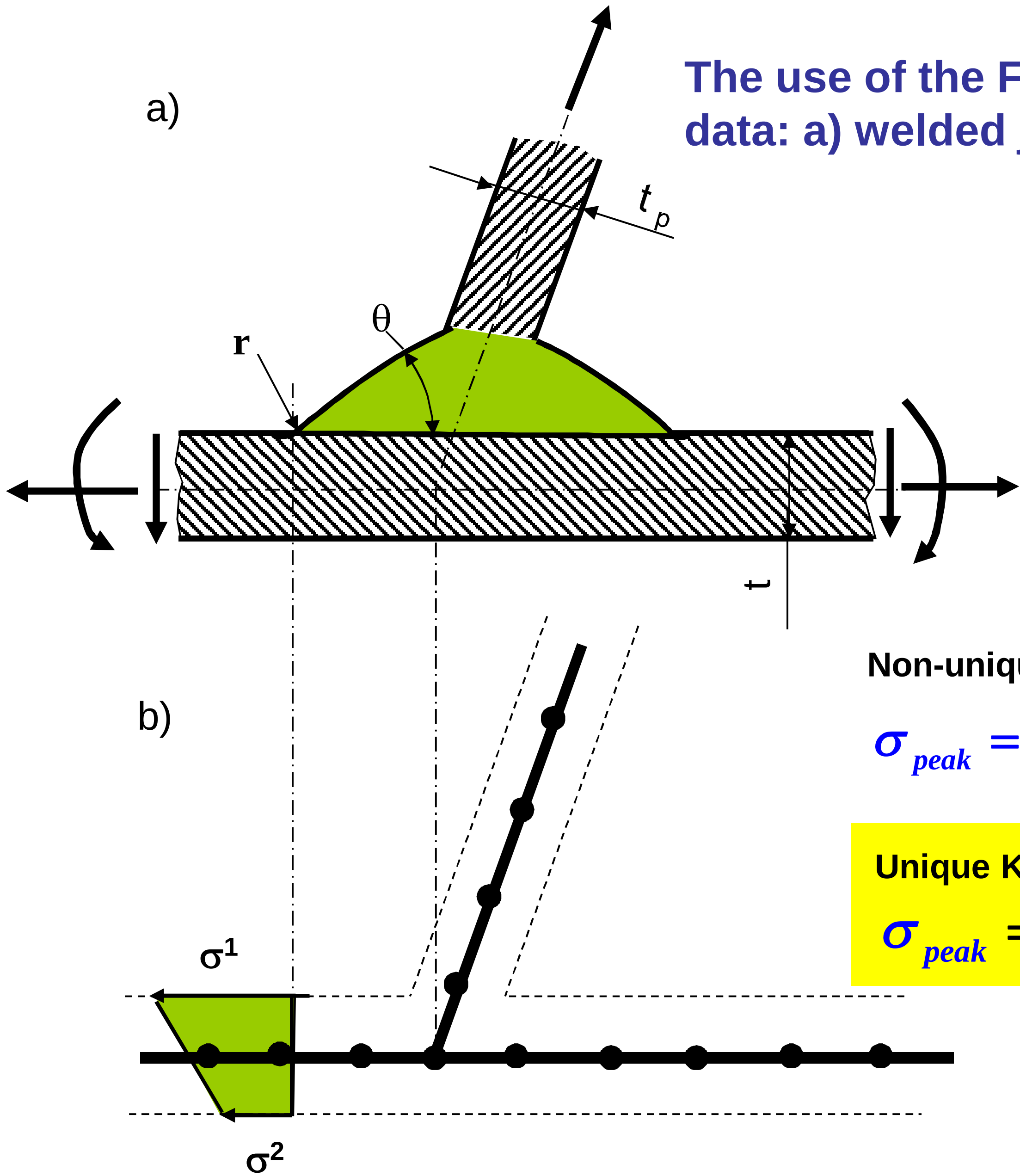
A shell finite element and the membrane, σ^{m}_{hs}, and bending, σ^{b}_{hs}, shell stresses

- The FE formulation for shell elements gives top and bottom stresses, σ_{top}, and σ_{bottom}
- The stress distribution through the thickness is considered to be linear
- The membrane and bending stresses are obtained from

$$\sigma^{m}_{hs} = \frac{\sigma_a + \sigma_b}{2} \qquad \sigma^{b}_{hs} = \frac{\sigma_a - \sigma_b}{2}$$



The use of the FE-shell stress analysis data: a) welded joint; b) shell FE model



$$\sigma_{hs} = \sigma_1 = \sigma_n$$

$$\sigma_{hs}^{m} = \frac{\sigma_1 + \sigma_2}{2}$$

$$\sigma_{hs}^{b} = \frac{\sigma_1 - \sigma_2}{2}$$

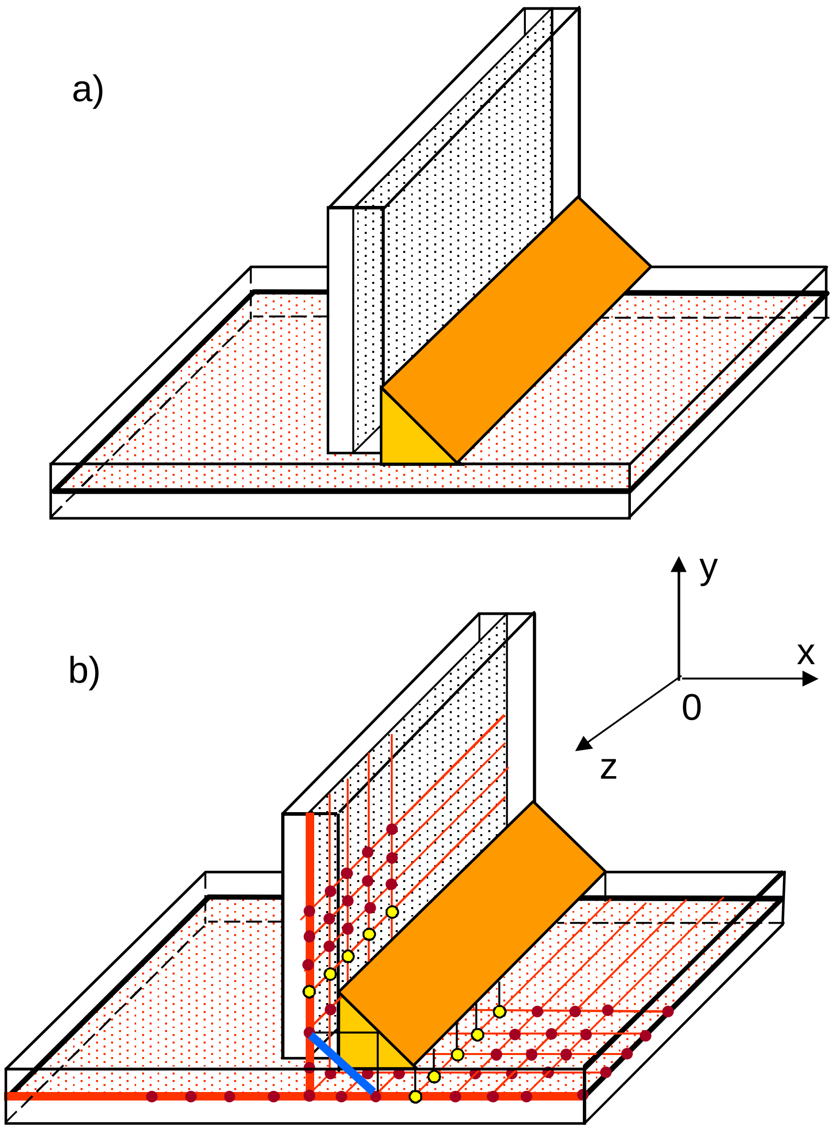
Non-unique K_t (depends on σ^m_{hs}/ σ^b_{hs})!

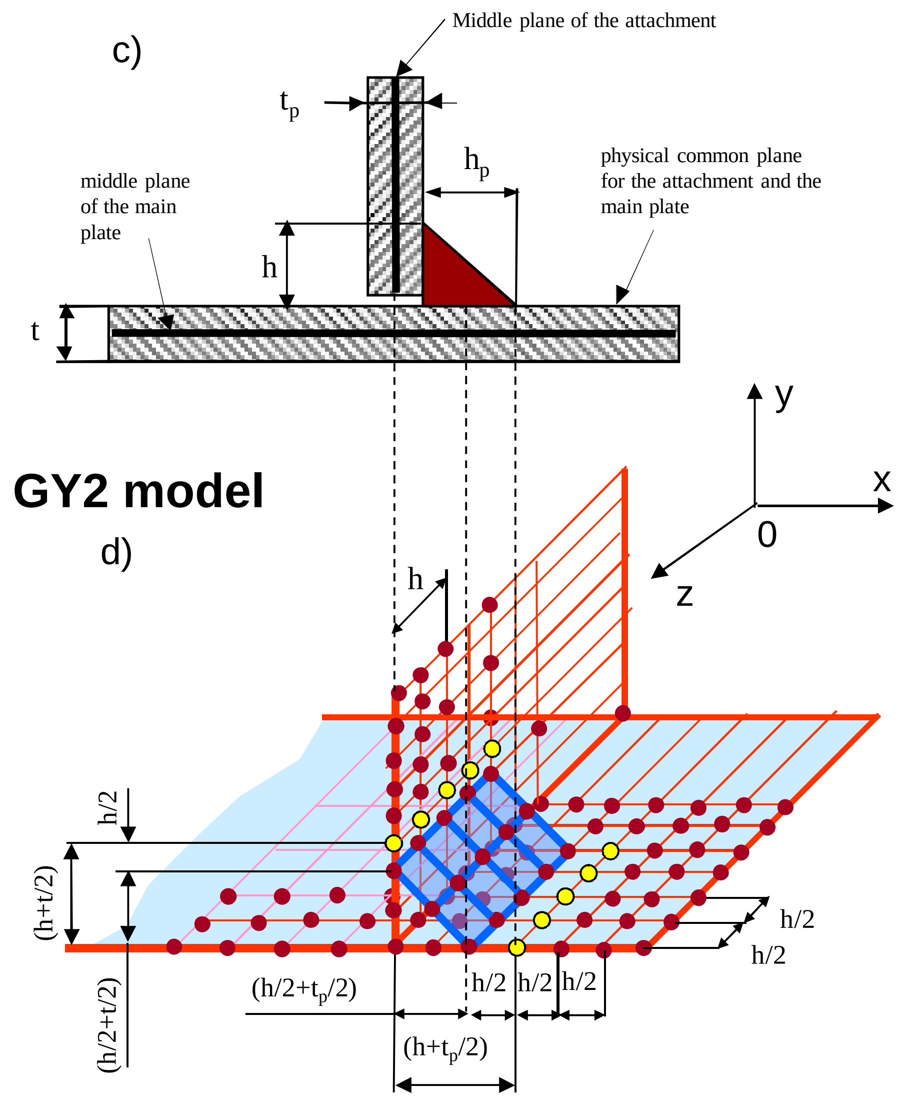
$$\sigma_{peak} = \sigma_{hs} K_t = \left(\sigma_{hs}^{m} + \sigma_{hs}^{b}\right) K_t$$

Unique K_t factors depend on geometry only!

$$\sigma_{peak} = K_{t,hs}^{m} \sigma_{hs}^{m} + K_{t,hs}^{b} \sigma_{hs}^{b}$$

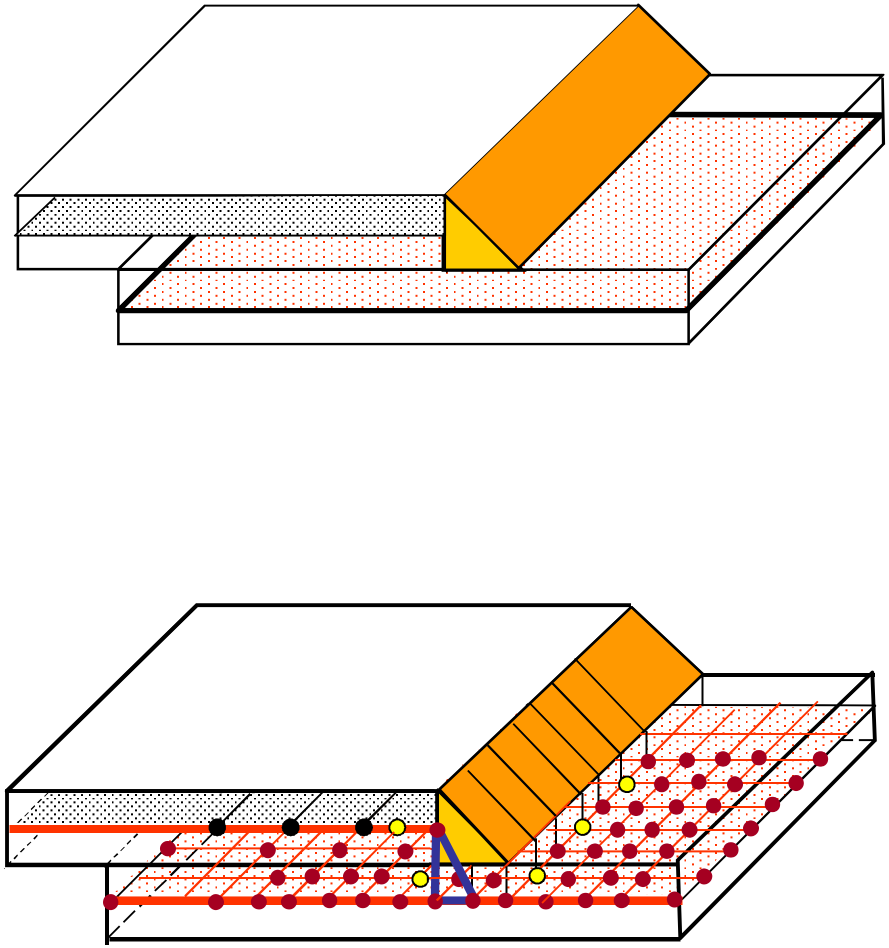
Single fillet weld without penetration; the shell FE model

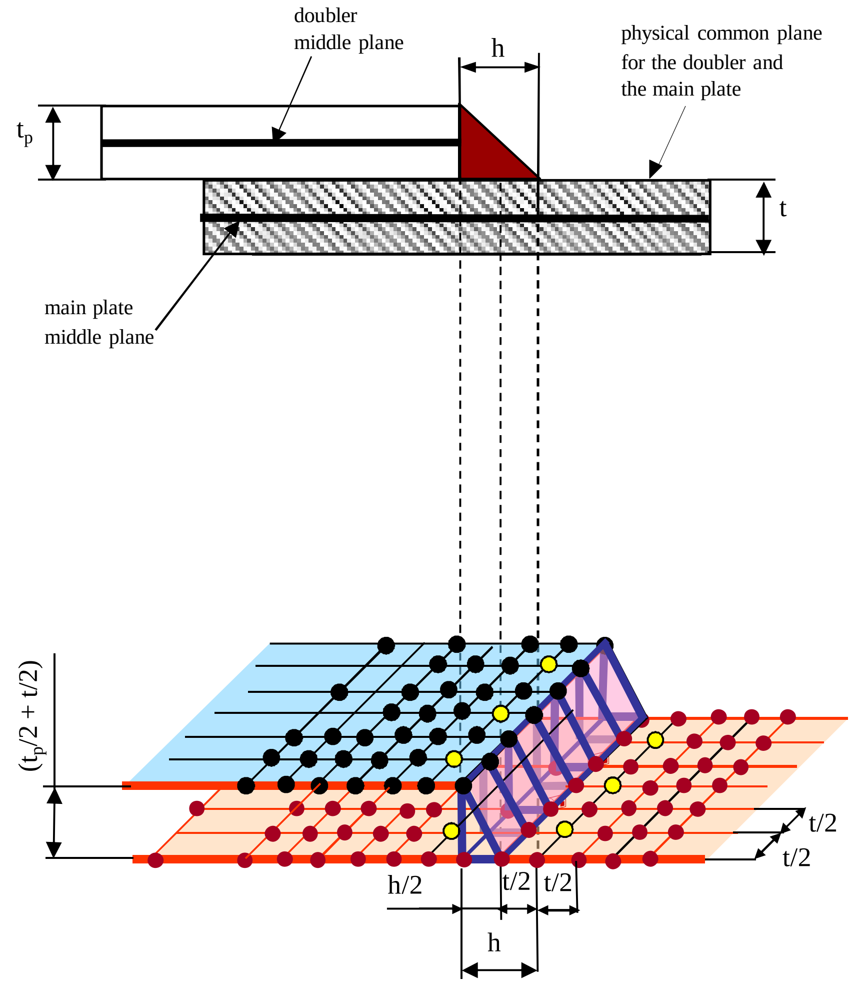




Note! The rectangles with blue edges are weld simulating shell elements with thickness equal to the thickness of the thinner plate.

Single fillet weld in double-overlap type configuration; FE model





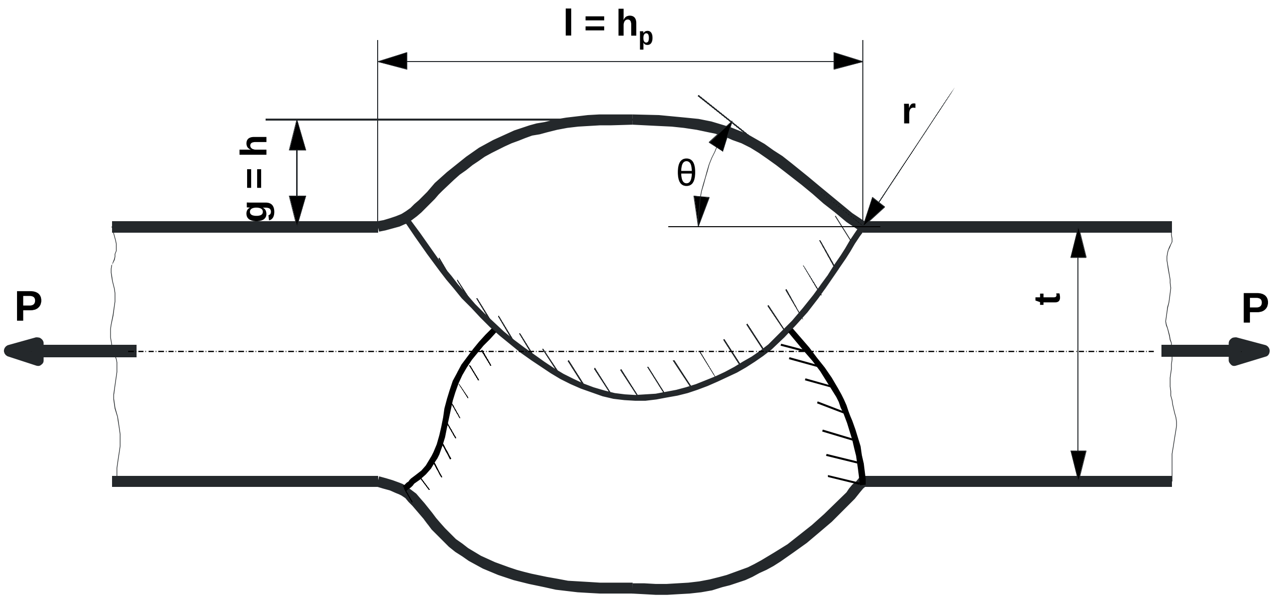
The “membrane, σ^m_{hs}, and bending, σ^b_{hs}, hot spot stresses” and the weld toe peak stress σ_{peak}

The advantage of using expression

$$\sigma_{peak} = K^m_{t,hs} \cdot \sigma^m_{hs} + K^b_{t,hs} \cdot \sigma^b_{hs}$$

lies in the fact that the membrane stress σ_{hs}^m and the bending stress σ_{hs}^b can be determined by simple decomposition of the linearized through-thickness stress field, ***σ(x=0,y)*,** which can be ***directly obtained from the coarse mesh 3-D or shell Finite Element (GY2) analysis***. Thus the equation above provides the link between the FE stress analysis data, σ_{hs}^m and σ_{hs}^b, and the peak stress, σ_{peak}, at the weld toe, necessary for the fatigue analysis

Stress concentration factor for a butt weldment under axial loading

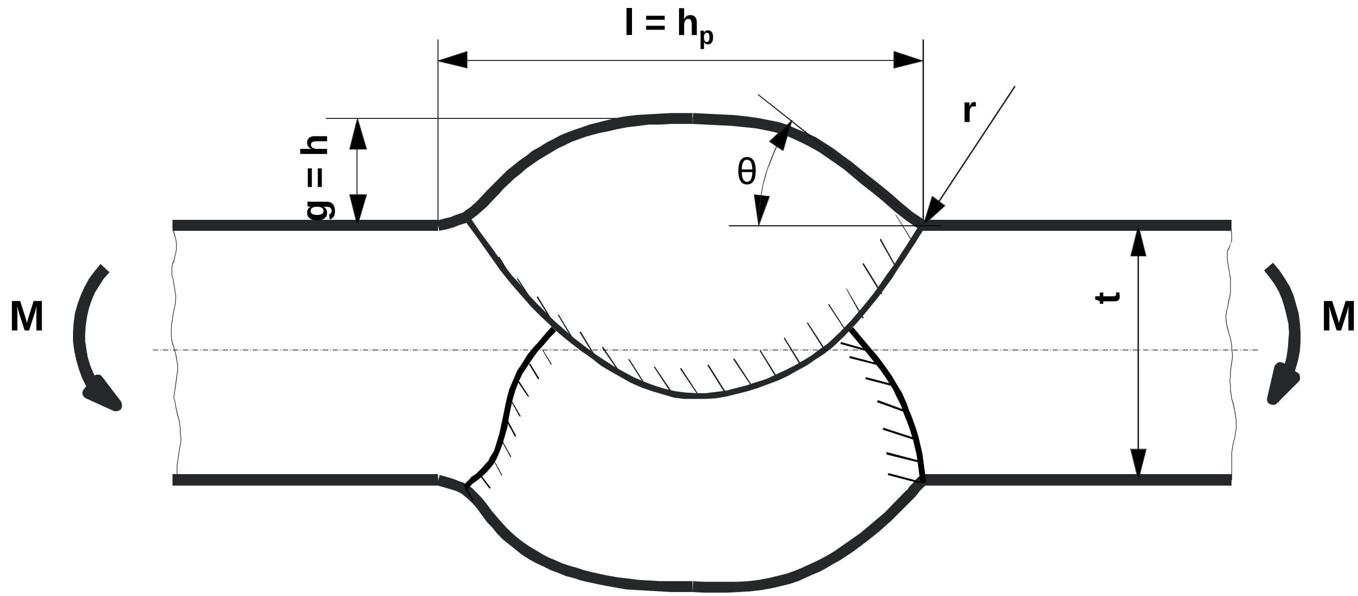


$$K_t^{ten} = 1 + \frac{1-\exp\left(-0.9\theta\sqrt{\frac{W}{2h}}\right)}{1-\exp\left(-0.45\pi\sqrt{\frac{W}{2h}}\right)} \times 2\left[\frac{1}{2.8\left(\frac{W}{t}\right)-2}\times\frac{h}{r}\right]^{0.65}$$

$$where:\ W = t + 2h + 0.6h_p$$

Range of application - reasonably designed weldments, (K.Iida and T. Uemura, ref. 14)

Stress concentration factor for a butt weldment under bending load



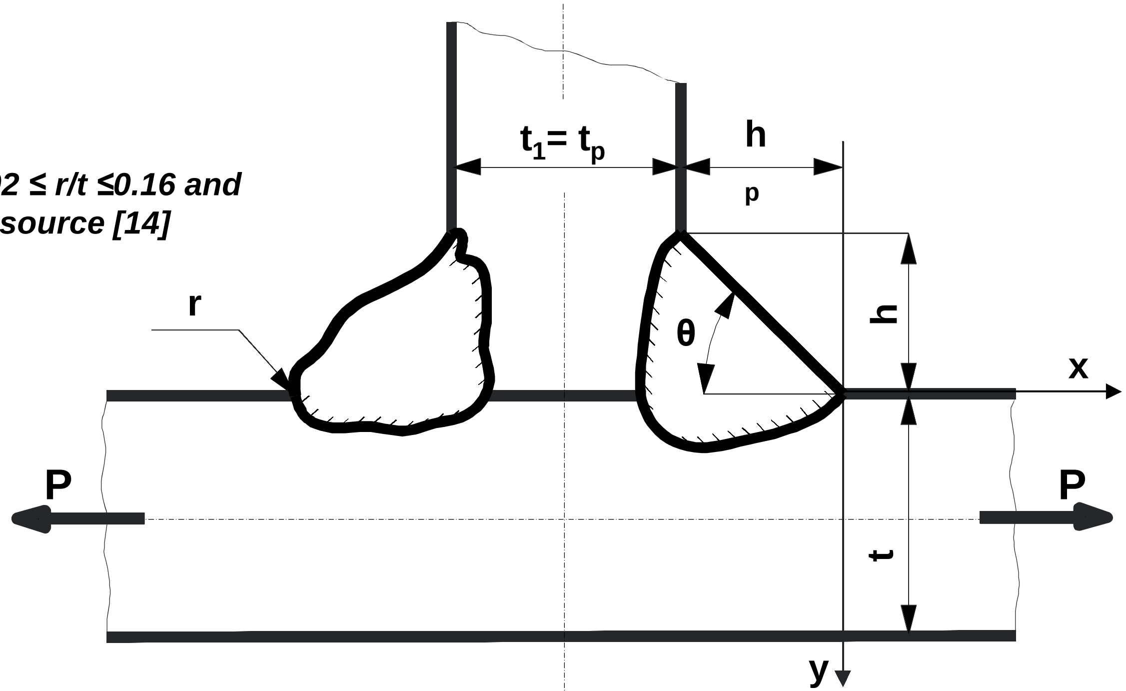
$$K_t^{ben} = 1 + \frac{1 - \exp\left(-0.9\theta\sqrt{\frac{W}{2h}}\right)}{1 - \exp\left(-0.45\pi\sqrt{\frac{W}{2h}}\right)} \times 1.5\sqrt{\tanh\left(\frac{2h_p}{1+2h} + \frac{2r}{t}\right)} \times \tanh\left[\frac{\left(\frac{2h}{t}\right)^{0.25}}{1 - \frac{r}{t}}\right] \times \left[\frac{0.13 + 0.65\left(1 - \frac{r}{t}\right)^4}{\left(\frac{r}{t}\right)^{\frac{1}{3}}}\right]$$

$$where: \quad W = t + 2h + 0.6h_p$$

Range of application - reasonably well designed weldments, *(K.Iida and T. Uemura, ref. 14)*

Stress concentration factor for a T-butt weldment under tension load; *(non-load carrying fillet weld)*

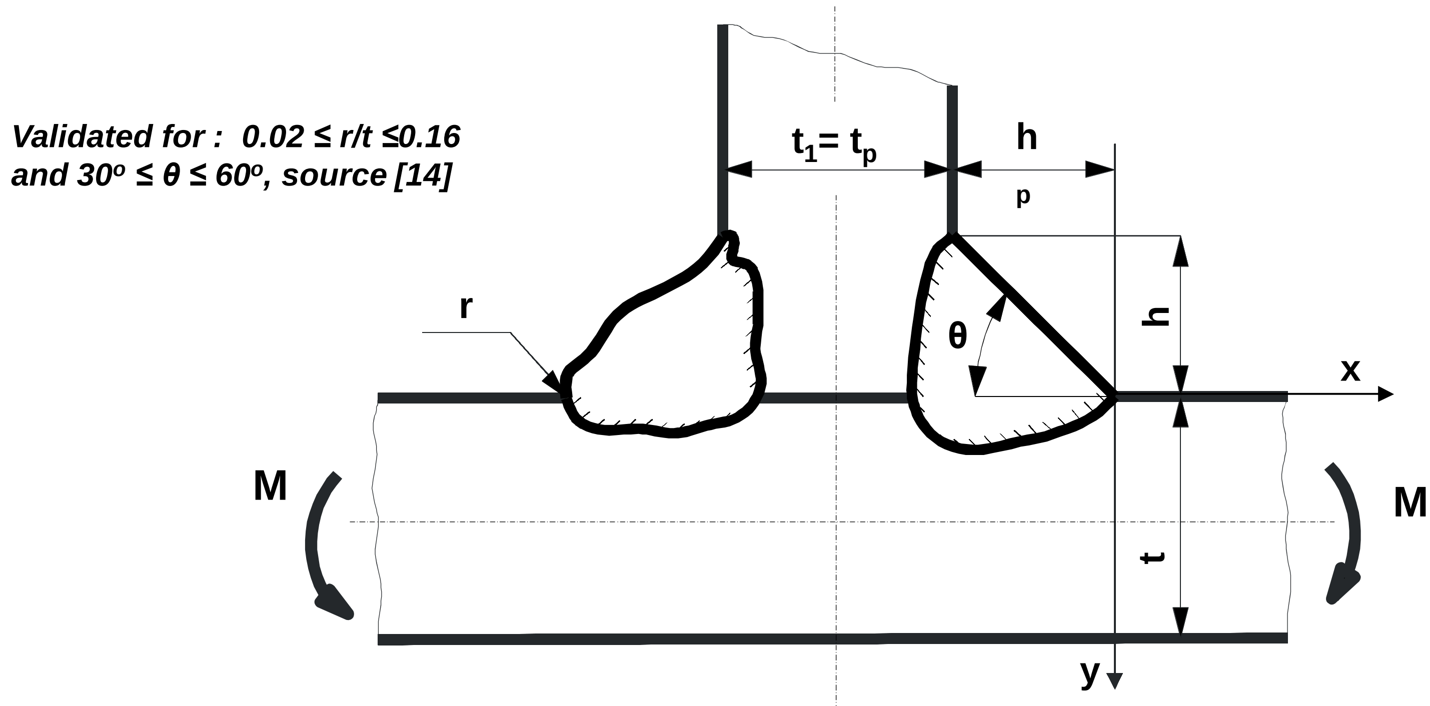
Validated for : 0.02 ≤ r/t ≤0.16 and
30º ≤ θ ≤ 60º, source [14]



$$K_t^t = 1 + \frac{1 - \exp\left(-0.9\theta\sqrt{\frac{W}{2h}}\right)}{1 - \exp\left(-0.45\pi\sqrt{\frac{W}{2h}}\right)} \times \left[\frac{1}{2.8\left(\frac{W}{t}\right) - 2} \times \frac{h}{r}\right]^{0.65}$$

$$where : W = (t + 2h) + 0.3(t_p + 2h_p)$$

Stress concentration factor for a T-butt weldment under bending load; *(non-load carrying fillet weld)*



$$K_t^b = 1 + \frac{1-\exp\left(-0.9\theta\sqrt{\frac{W}{2h}}\right)}{1-\exp\left(-0.45\pi\sqrt{\frac{W}{2h}}\right)} \times 1.9\sqrt{\tanh\left(\frac{2t_p}{t+2h}+\frac{2r}{t}\right)} \times \tanh\left[\frac{\left(\frac{2h}{t}\right)^{0.25}}{1-\frac{r}{t}}\right] \times \left[\frac{0.13+0.65\left(1-\frac{r}{t}\right)^4}{\left(\frac{r}{t}\right)^{\frac{1}{3}}}\right];$$

where: $W = (t+2h)+0.3(t_p+2h_p)$

T-butt weldment subjected to *pure tension*;

Monahan's equation for the dominant stress component over the entire critical cross section

$$\sigma(y) = \frac{K_t^t \sigma_n}{2\sqrt{2}} \left[\left(\frac{y}{r} + \frac{1}{2} \right)^{-\frac{1}{2}} + \frac{1}{2} \left(\frac{y}{r} + \frac{1}{2} \right)^{-\frac{3}{2}} \right] \frac{1}{G_m}$$

Where:

$$K_t^t = 1 + 0.388 \times \theta^{0.37} \left(\frac{r}{t} \right)^{-0.454}$$

$$G_m = 1 \quad for \quad \frac{y}{r} \leq 0.3$$

$$G_m = 0.06 + \frac{0.94 \times e^{-E_m T_m}}{1 + E_m^3 T_m^{0.8} \times e^{-E_m T_m^{1.1}}} \quad for \quad \frac{y}{r} > 0.3$$

$$E_m = 1.05 \times \theta^{0.18} \left(\frac{r}{t} \right)^{q}$$

$$q = -0.12\,\theta^{-0.62}$$

$$T_m = \frac{y}{t} - 0.3 \frac{r}{t}.$$

Derived for: $\frac{\pi}{6} \leq \theta \leq \frac{\pi}{3} \quad and \quad \frac{1}{50} \leq \frac{r}{t} \leq \frac{1}{15} \quad and \quad 0 \leq y \leq t$

T-butt weldment subjected to *pure bending*;

Monahan's equation for the dominant stress component over the entire critical cross section

$$\sigma(y) = \frac{K_t^b \sigma_n}{2\sqrt{2}} \left[\left(\frac{y}{r} + \frac{1}{2} \right)^{-\frac{1}{2}} + \frac{1}{2} \left(\frac{y}{r} + \frac{1}{2} \right)^{-\frac{3}{2}} \right] \frac{1 - 2\left(\frac{y}{t}\right)}{G_b}$$

Where:

$$K_t^b = 1 + 0.512 \times \theta^{0.572} \left(\frac{r}{t} \right)^{-0.469}$$

$$G_b = 1 \quad for \quad \frac{y}{r} \leq 0.4$$

$$G_b = 0.07 + \frac{0.93 \times e^{-E_b T_b}}{1 + E_b^3 T_b^{0.6} \times e^{-E_b T_b^{1.2}}} \quad for \quad \frac{y}{r} > 0.4$$

$$E_b = 0.9 \left(\frac{r}{t} \right)^{-\left(0.0026 + \frac{0.0825}{\theta} \right)}$$

$$T_b = \frac{y}{t} - 0.4 \frac{r}{t}.$$

Derived for: $\frac{\pi}{6} \leq \theta \leq \frac{\pi}{3} \quad and \quad \frac{1}{50} \leq \frac{r}{t} \leq \frac{1}{15} \quad and \quad 0 \leq y \leq t$

Theoretical through-thickness stress distribution

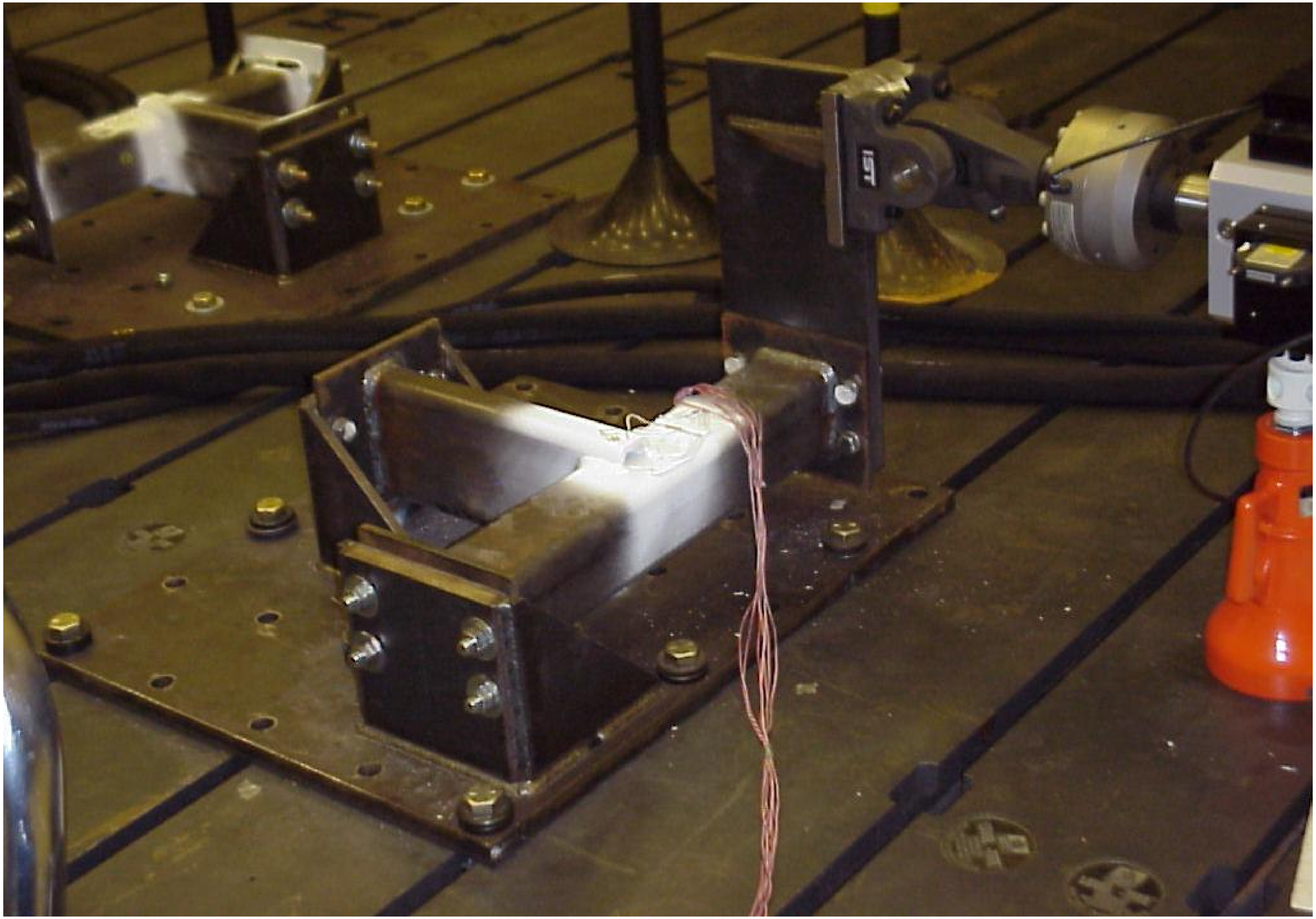
(Monahan's equations for mixed mode loading, i.e. simultaneous axial and bending)

$$\sigma\left(x=0,y\right)^{m}=\frac{K_t^{t}\sigma_{hs}^{m}}{2\sqrt{2}}\left[\left(\frac{y}{r}+\frac{1}{2}\right)^{-\frac{1}{2}}+\frac{1}{2}\left(\frac{y}{r}+\frac{1}{2}\right)^{-\frac{3}{2}}\right]\frac{1}{G_m}$$

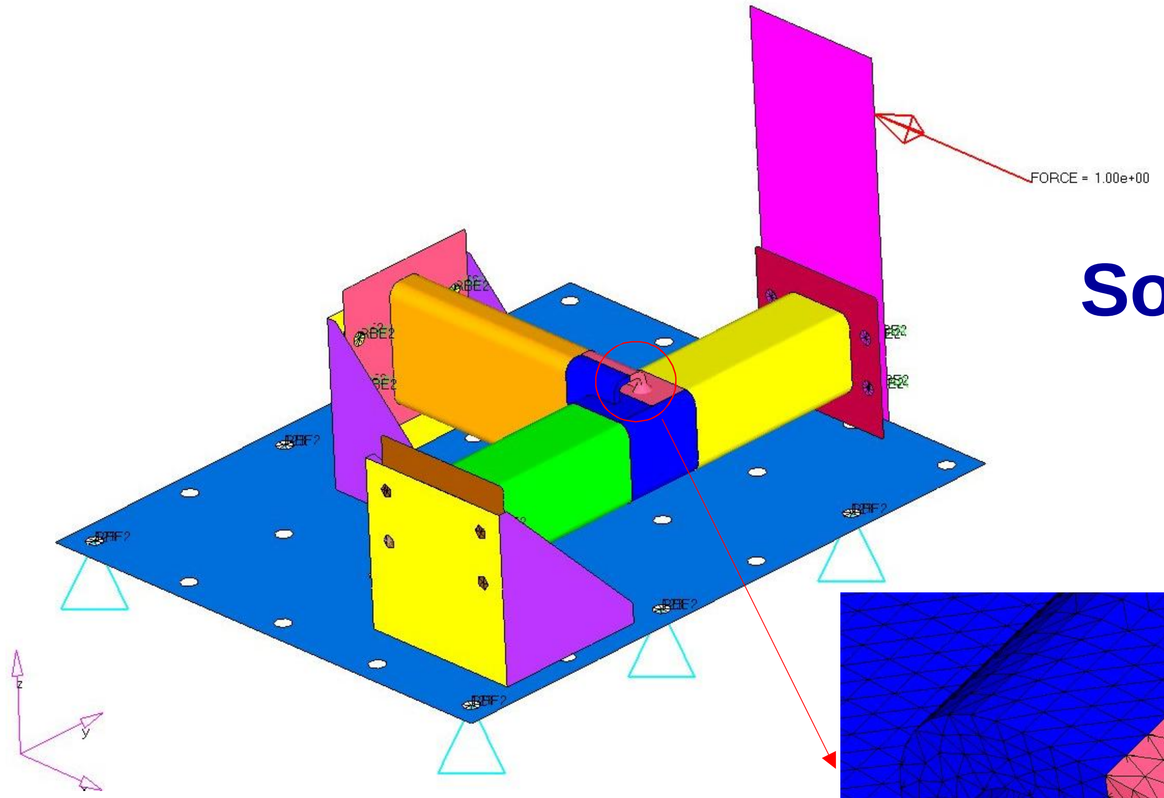
$$\sigma\left(x=0,y\right)^{b}=\frac{K_t^{b}\sigma_{hs}^{b}}{2\sqrt{2}}\left[\left(\frac{y}{r}+\frac{1}{2}\right)^{-\frac{1}{2}}+\frac{1}{2}\left(\frac{y}{r}+\frac{1}{2}\right)^{-\frac{3}{2}}\right]\frac{1-2\left(\frac{y}{t}\right)}{G_b}$$

$$\sigma\left(y\right)=\sigma\left(y\right)^{m}+\sigma\left(y\right)^{b}=\left[\frac{K_t^{m}\sigma_{hs}^{m}}{2\sqrt{2}}\cdot\frac{1}{G_m}+\frac{K_t^{b}\sigma_{hs}^{b}}{2\sqrt{2}}\cdot\frac{1-2\left(\frac{y}{t}\right)}{G_b}\right]\left[\left(\frac{y}{r}+\frac{1}{2}\right)^{-\frac{1}{2}}+\frac{1}{2}\left(\frac{y}{r}+\frac{1}{2}\right)^{-\frac{3}{2}}\right]$$

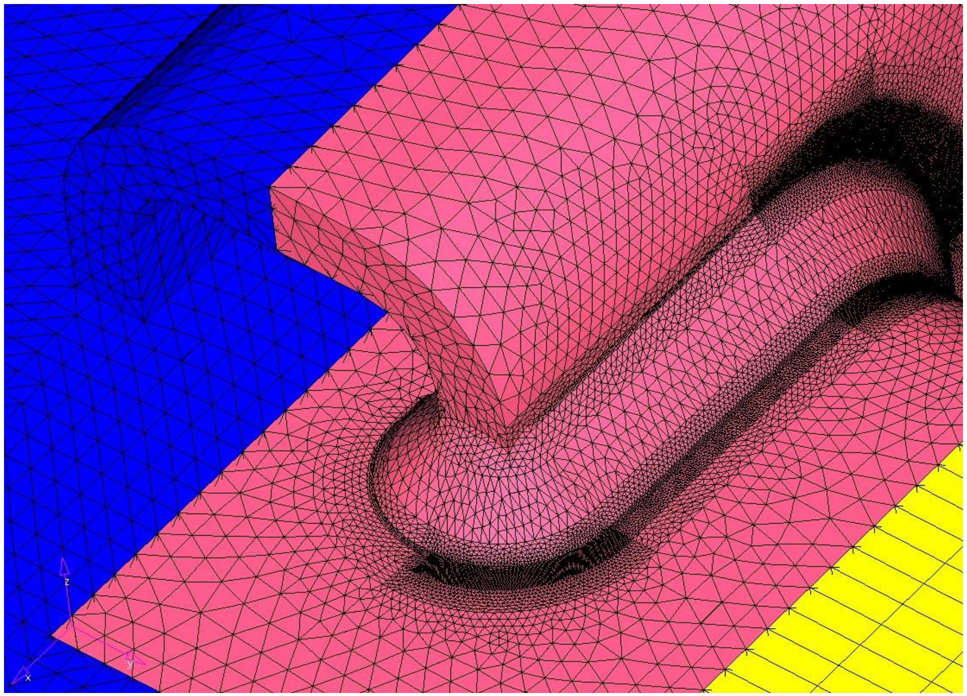
Tubular Welded Joint under Torsion and Bending



Courtesy of John Deere Co.

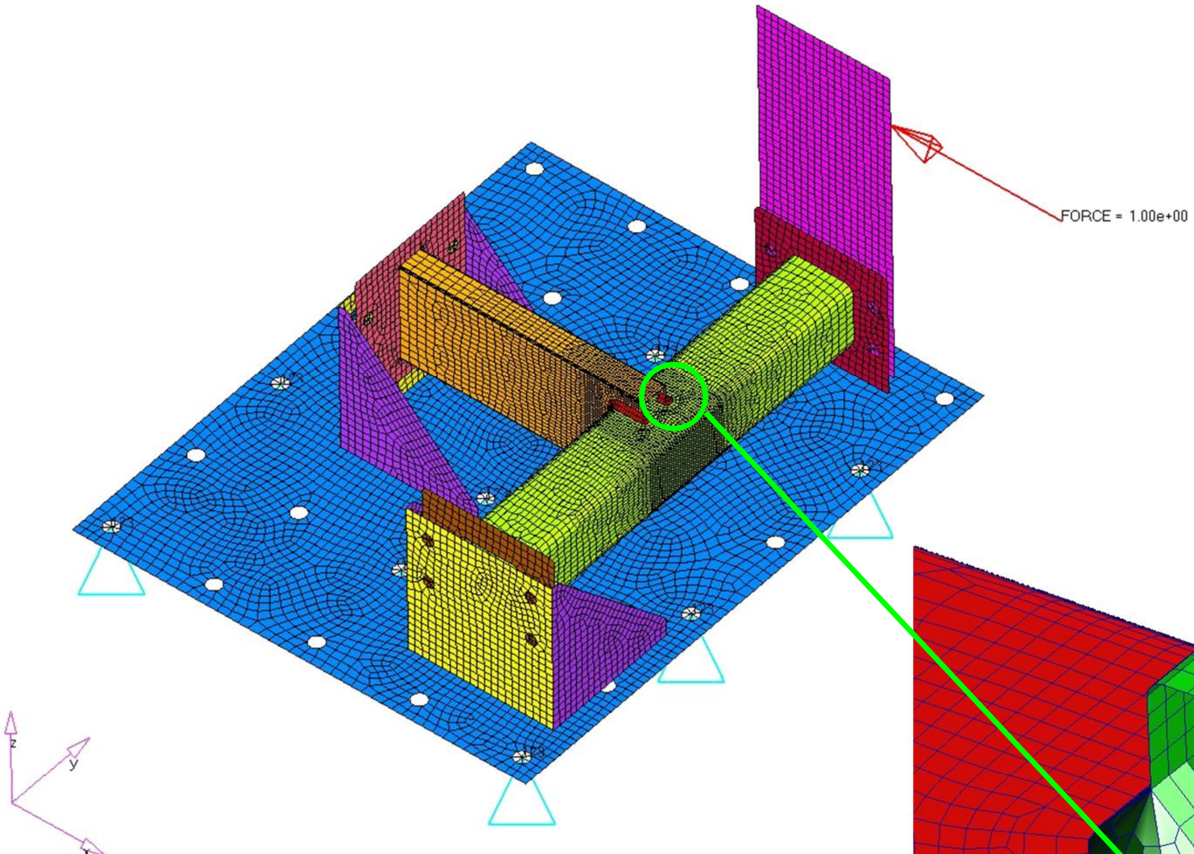


Solid and FE model

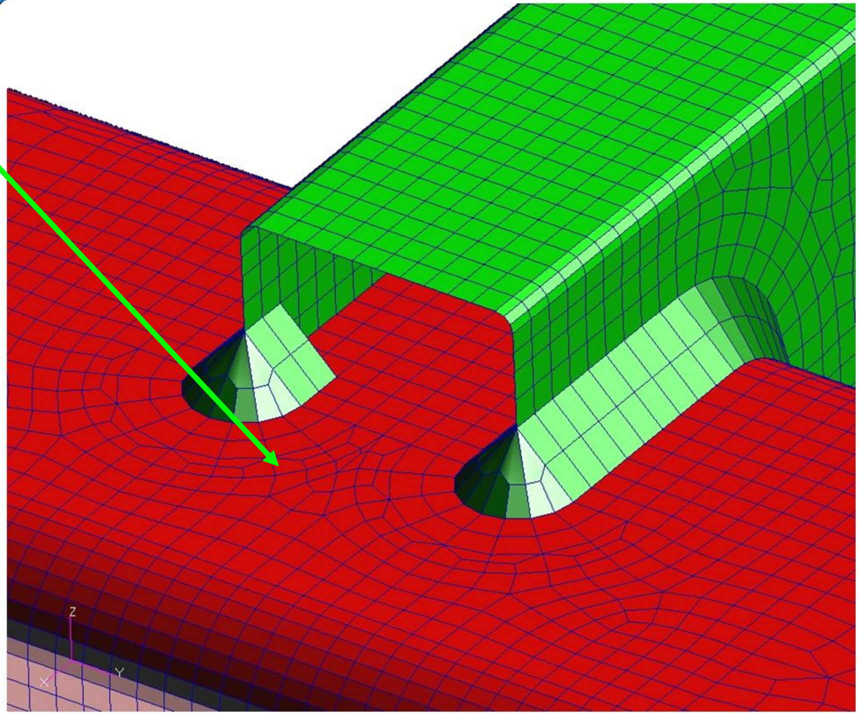


Courtesy of John Deere Co.

Shell Element Model Details

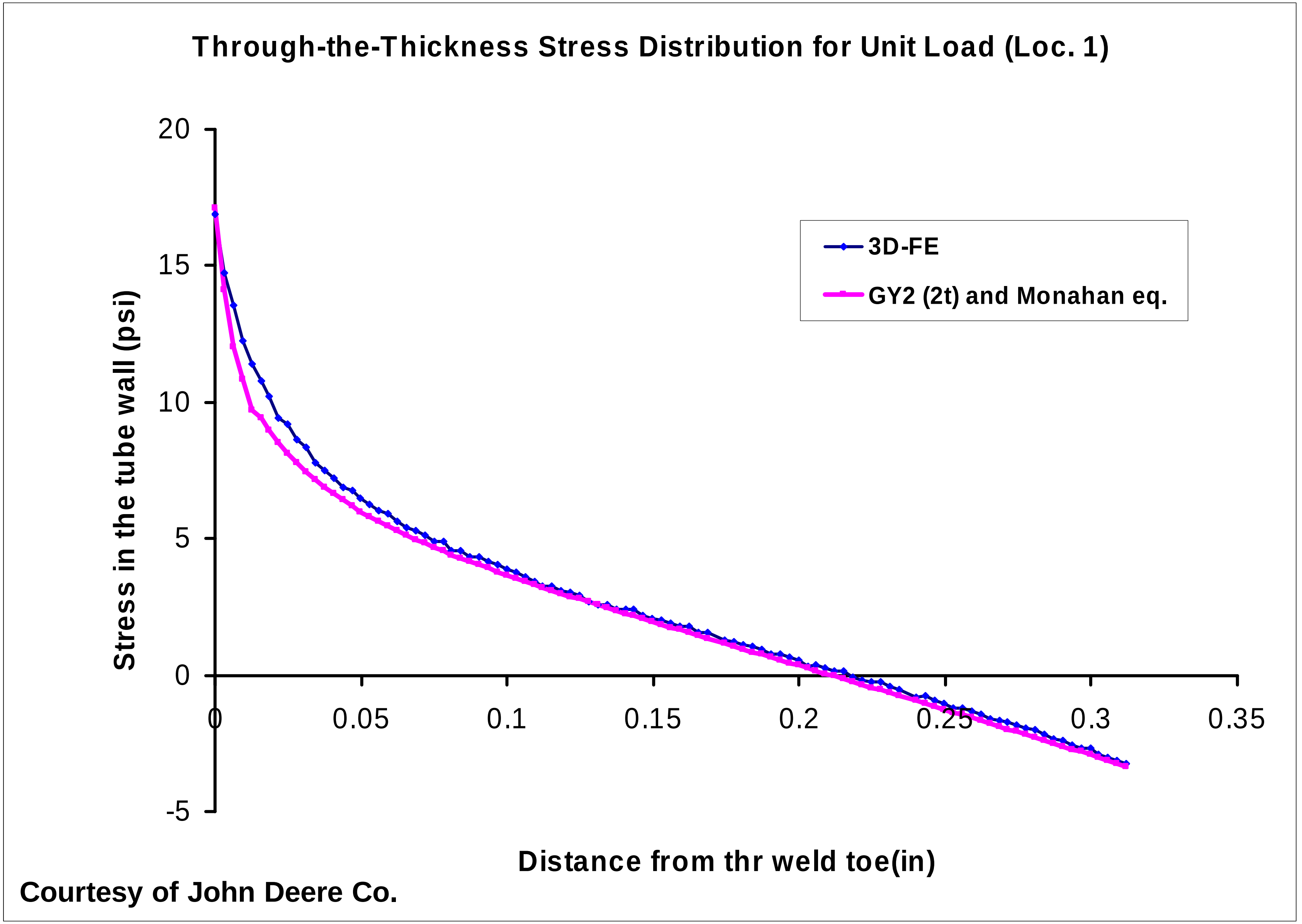


19197 nodes
18858 elements (linear quads)
114069 dof
Follows GY-2 modeling practice



Material:
A22H Steel (ASTM A500 Cold Formed Steel for Structural Tubing)

Courtesy of John Deere Co.

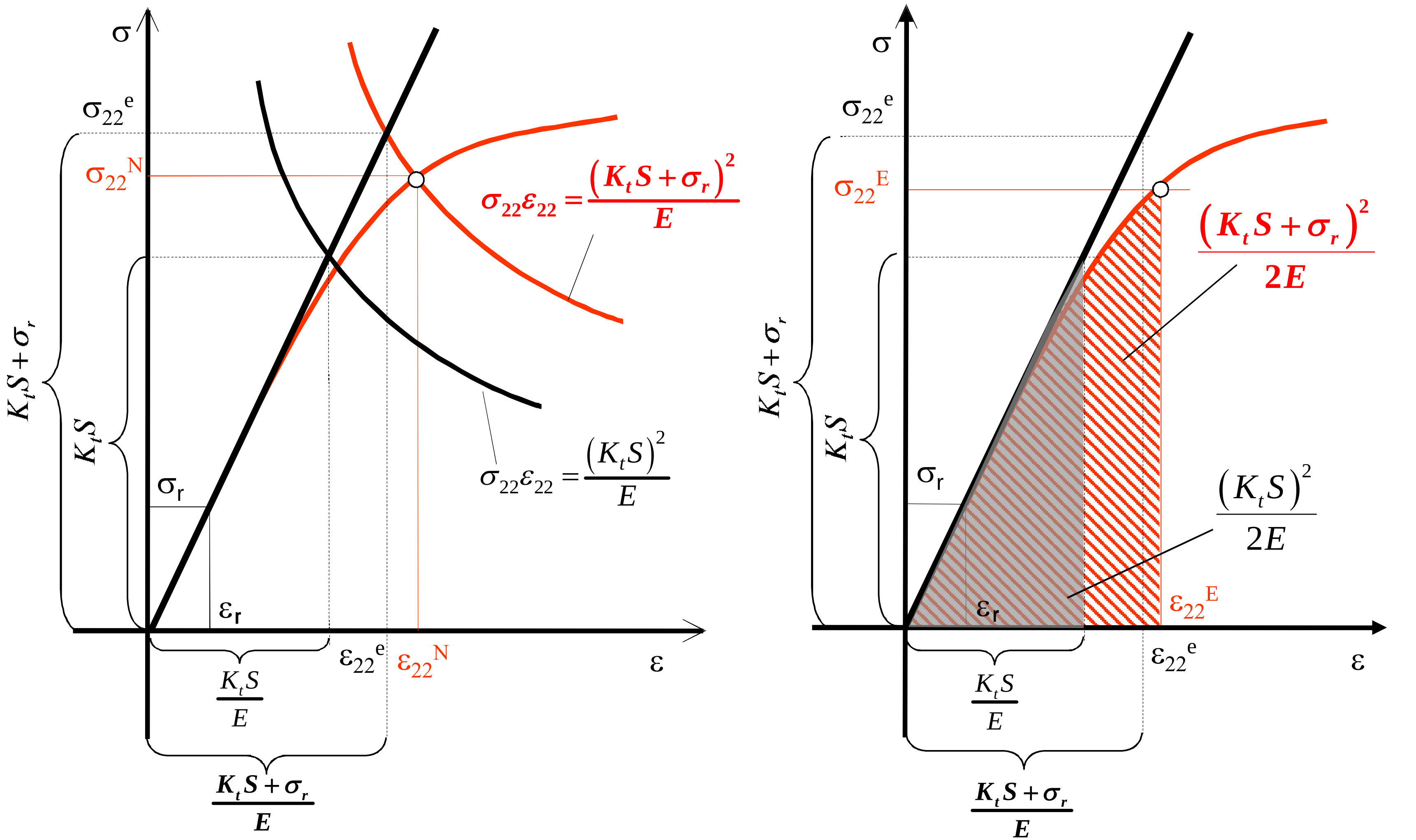


Through-the-Thickness Stress Distribution for Unit Load (Loc. 1)
Stress in the tube wall (psi)
20
15
10
5
0
-5
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
3D-FE
GY2 (2t) and Monahan eq.
Distance from thr weld toe(in)
Courtesy of John Deere Co.

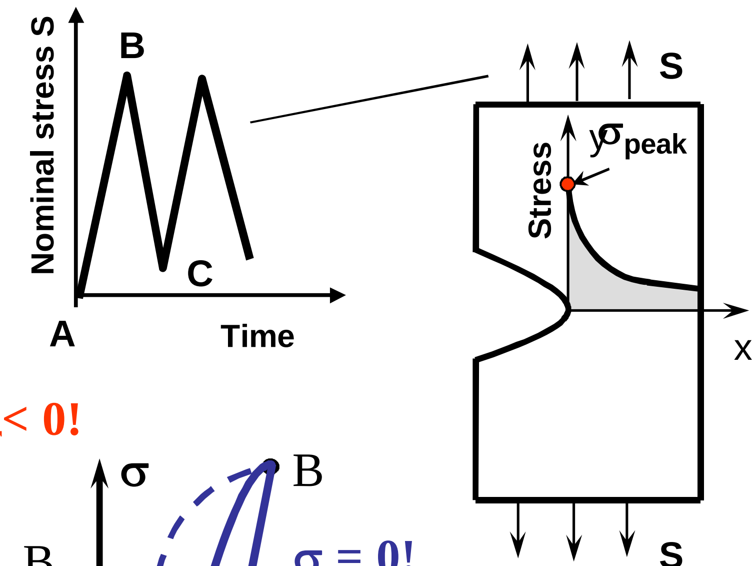
Modeling of the residual stress effect

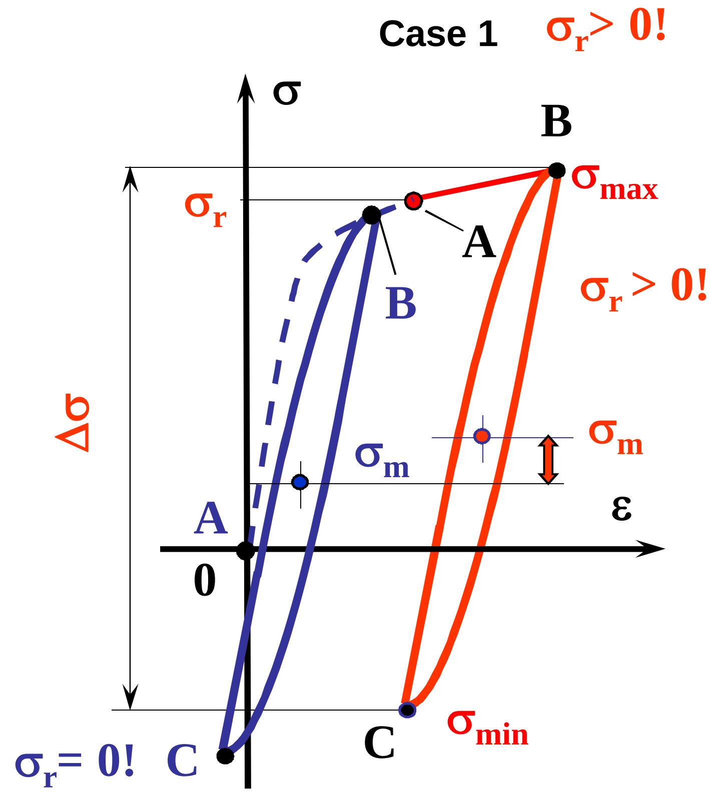
$$\frac{(K_t S+\sigma_r)^2}{E}=\sigma_{22}^N \varepsilon_{22}^N$$ - Neuber's rule

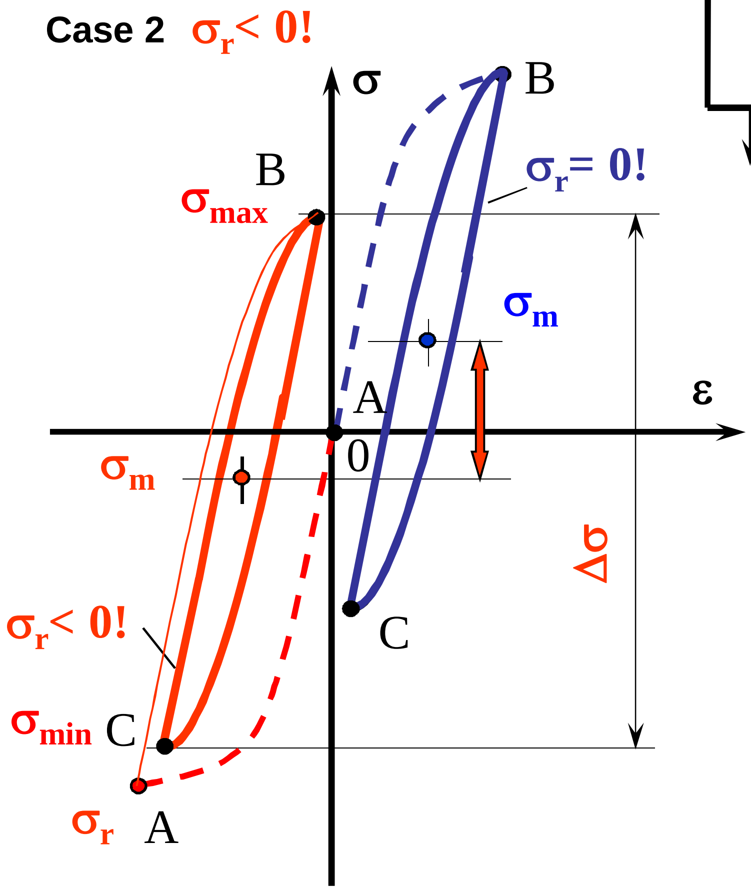
$$\frac{(K_t S+\sigma_r)^2}{2E}=\int_0^{\varepsilon_{22}^E} \sigma_{22}^E d\varepsilon_{22}^E$$ - ESED method



Residual Stress Effect on the Stress-Strain Response at the Notch Tip







Example:

Two plates A and B are connected by a double–sided butt weld. Another plate C is welded to plates A by fillet welds as shown in the Figure below. The plate is subjected to cyclic loading with a constant stress range of ΔS_t = 80MPa. It is assumed that the fabrication meets the standard requirements which allow the maximum misalignments of the butt weld to be "e" = 3 mm.

- Where are fatigue cracks most likely to be expected?
- What is the expected fatigue life of the joint?

Material: welded steel

σ_{ys} = 232 MPa, σ_{uts} = 414MPa,

E = 190000 MPa

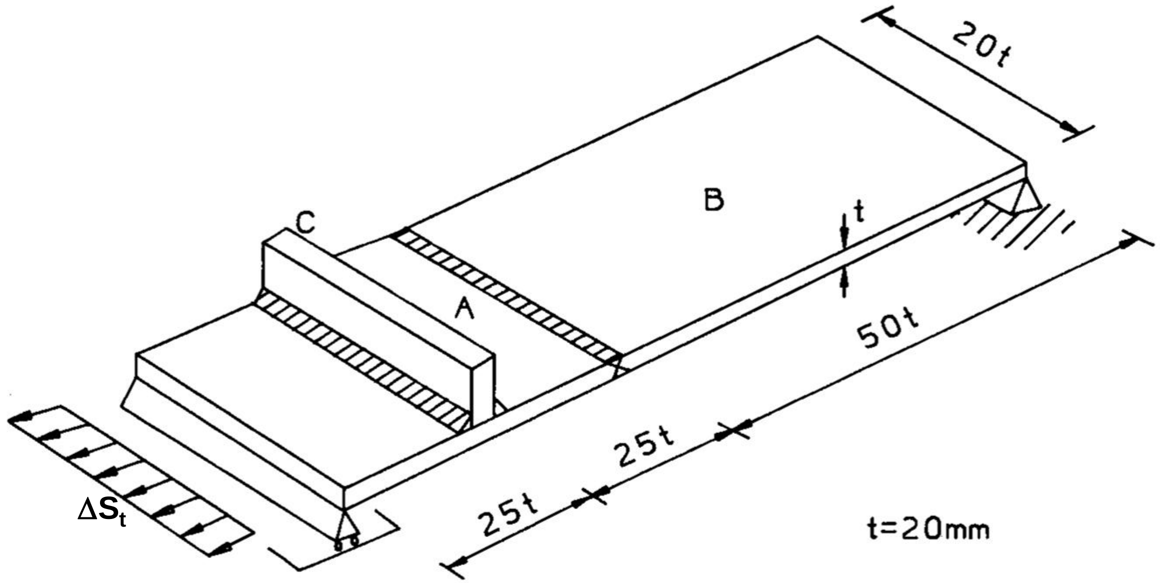
K' = 1097 MPa, n' = 0.249

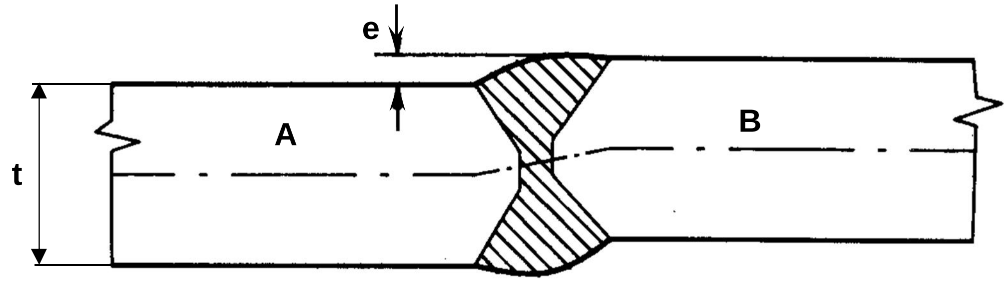
σ_f' = 1014MPa, b= -0.132

ε_f' = 0.271, c = -0.451

$\theta = 18^0$ - butt weld, weld toe radius, r = 0.8 mm, t = 20mm

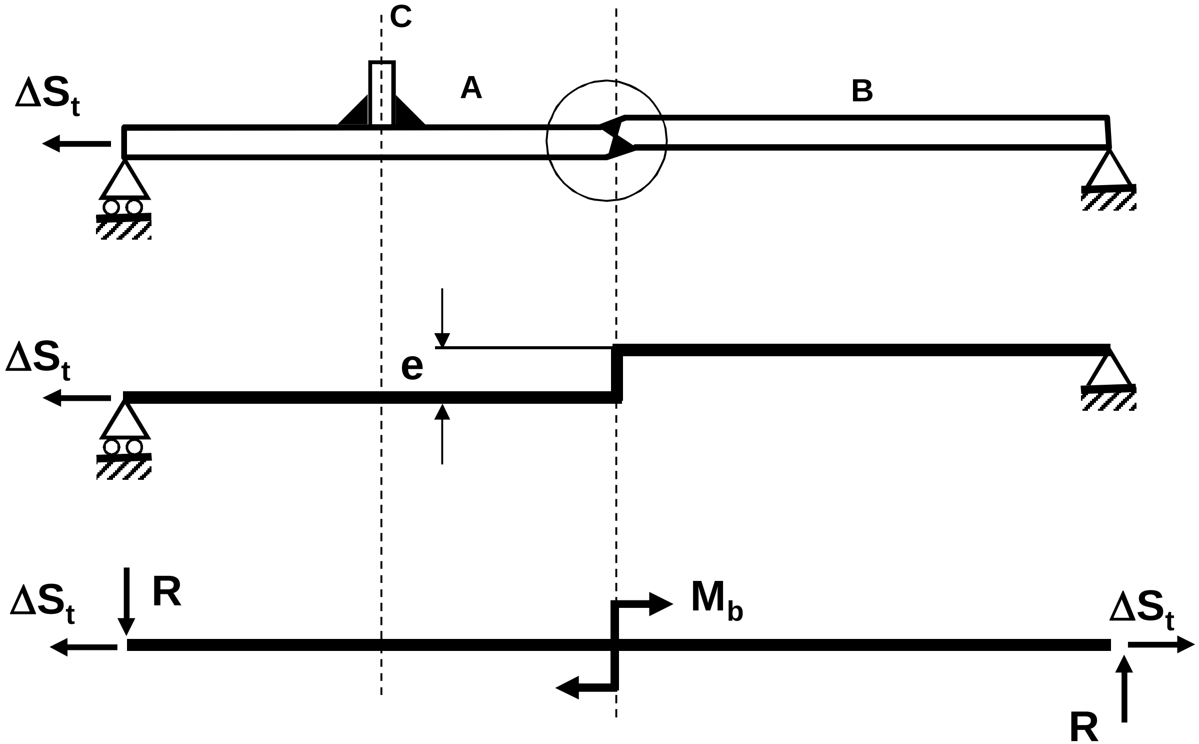
$\theta = 45^0$ - fillet weld





- bending moment at the butt weld: $\Delta M_I = 10 (\Delta S_t)(t^2 \bullet e)$

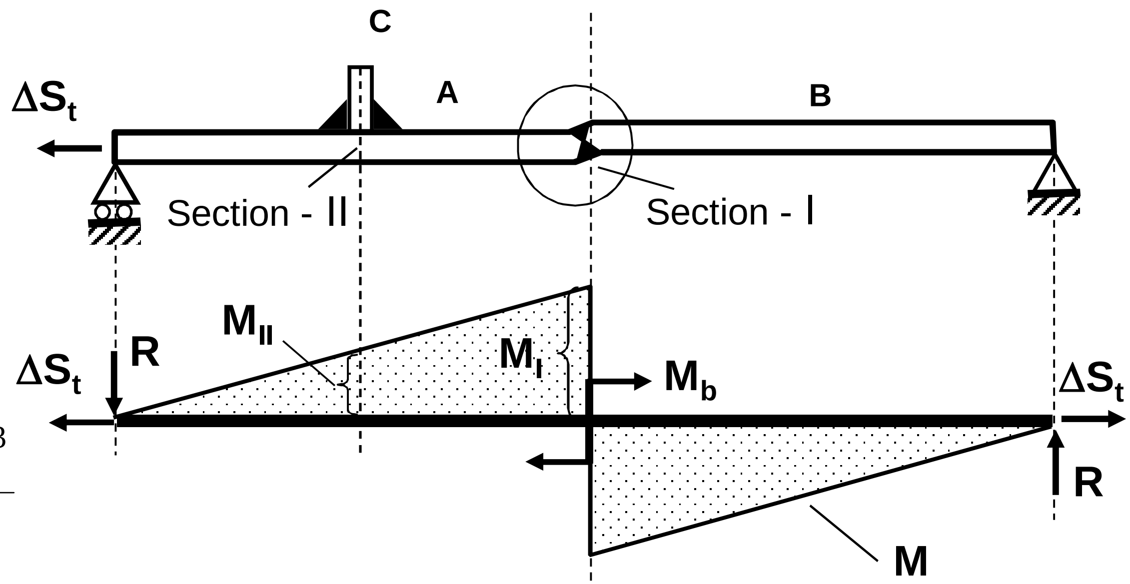
- bending moment at fillet welds: $\Delta M_I = 5 (\Delta S_t)(t^2 \bullet e)$



$$M_b = \Delta S_t \cdot b \cdot t \cdot e$$

$$R \cdot L - M_b = 0$$

$$R = \frac{M_b}{L} = \frac{\Delta S_t \cdot b \cdot t \cdot e}{L} = \frac{\Delta S_t \cdot 20t \cdot t \cdot e}{100t} = \frac{\Delta S_t \cdot t \cdot e}{5}$$



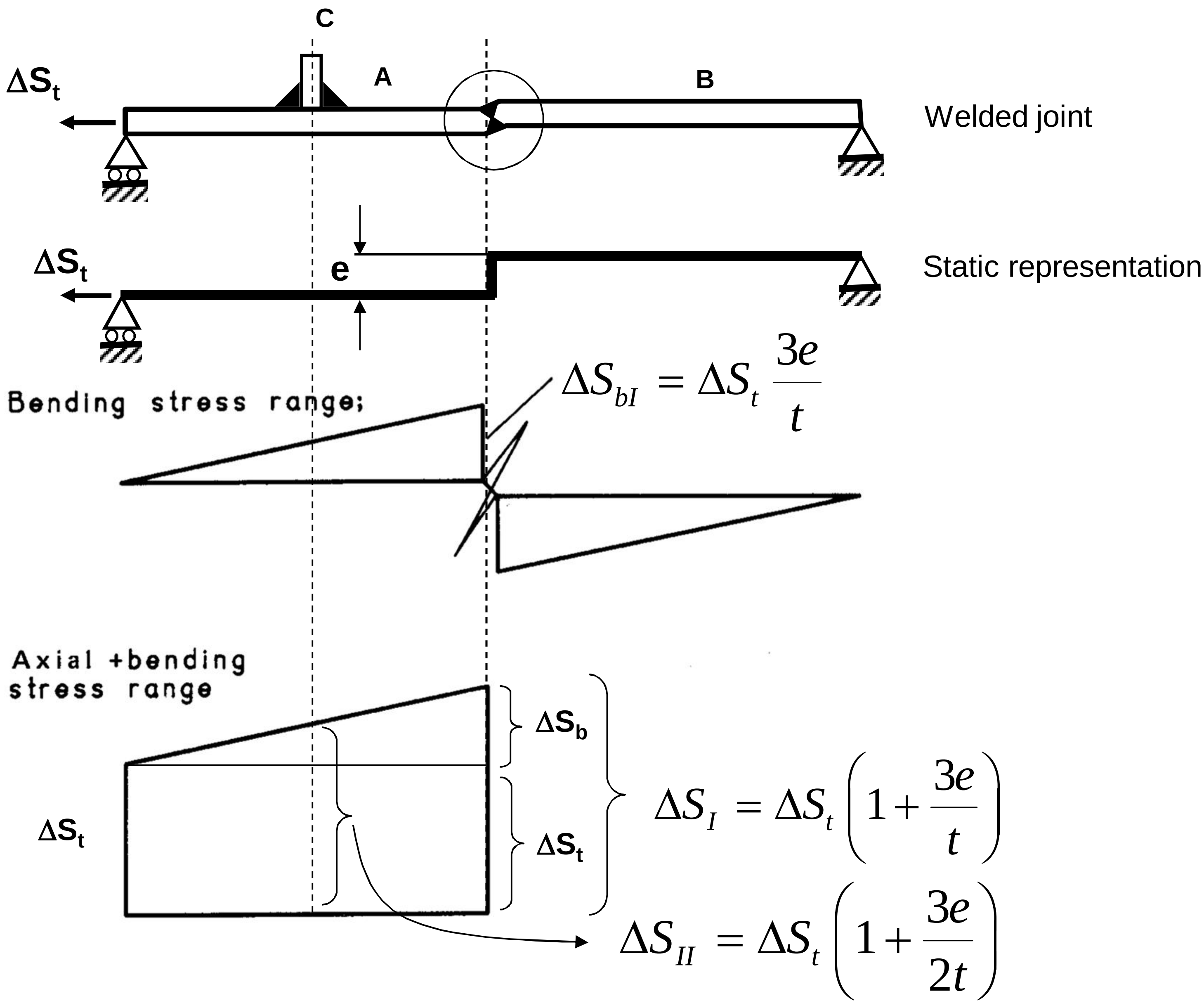
$$W = \frac{\dfrac{bt^3}{12}}{\dfrac{t}{2}} = \frac{bt^2}{6} = \frac{20t^3}{6}$$

$$M_I = R \cdot 50t = \frac{\Delta S_t \cdot t \cdot e}{5} \cdot 50t = 10 \cdot \Delta S_t \cdot t^2 \cdot e$$

$$M_{II} = R \cdot 75t - M_b = \frac{\Delta S_t \cdot t \cdot e}{5} \cdot 75t - \Delta S_t \cdot 20t \cdot t \cdot e = -5\Delta S_t \cdot t^2 \cdot e$$

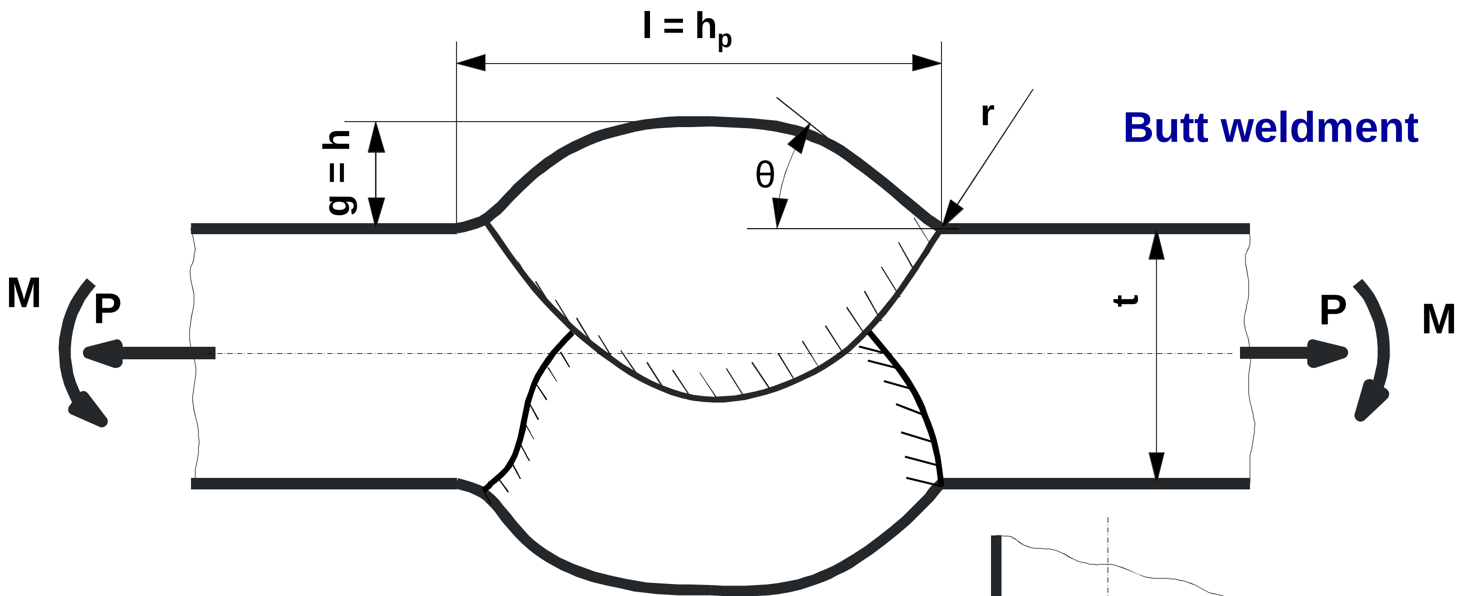
$$\Delta S_I = \Delta S_t + \Delta S_{bI} = \Delta S_t + \frac{M_I}{W} = \Delta S_t + \frac{10\Delta S \cdot t^2 \cdot e}{20t^3 / 6} = \Delta S_t \left(1 + \frac{3e}{t}\right)$$

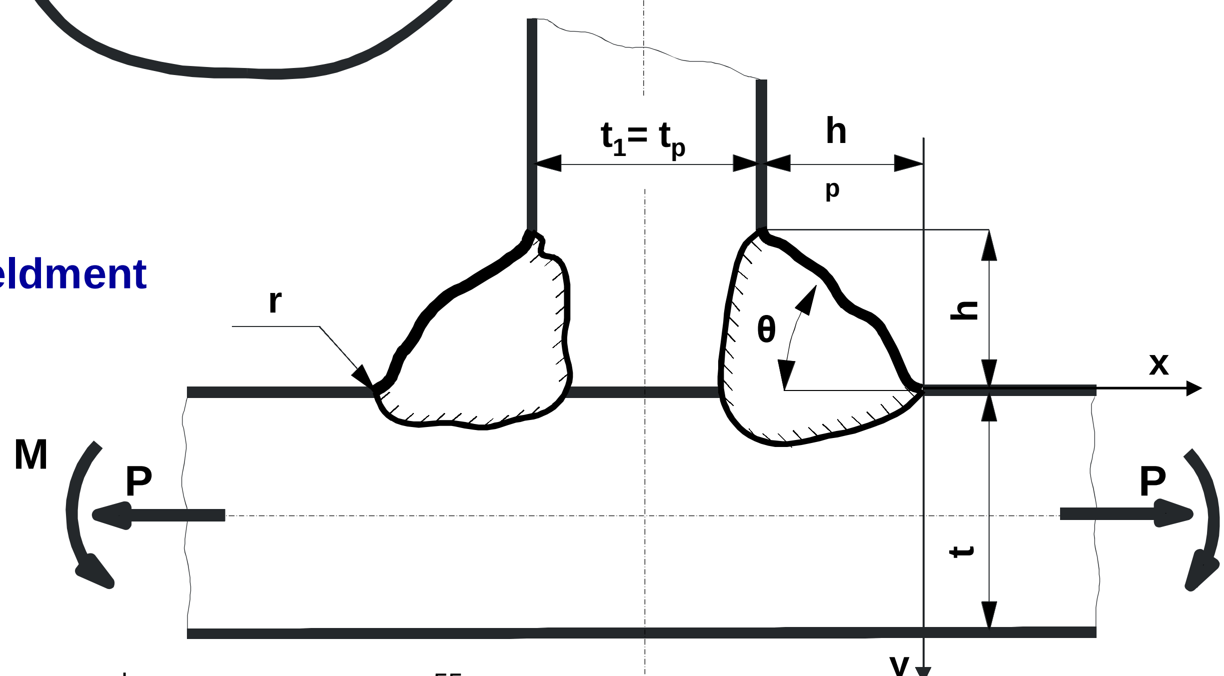
$$\Delta S_{II} = \Delta S_t + \Delta S_{bII} = \Delta S_t + \frac{M_{II}}{W} = \Delta S_t + \frac{5\Delta S \cdot t^2 \cdot e}{20t^3 / 6} = \Delta S_t \left(1 + \frac{3e}{2t}\right)$$



C
A
B
ΔS_t
Welded joint
ΔS_t
e
Static representation
Bending stress range;
$\Delta S_{bI} = \Delta S_t \frac{3e}{t}$
Axial +bending
stress range
ΔS_b
ΔS_t
ΔS_t
$\Delta S_I = \Delta S_t \left(1 + \frac{3e}{t}\right)$
$\Delta S_{II} = \Delta S_t \left(1 + \frac{3e}{2t}\right)$

Stress Concentration Factor for Butt and T-Butt Weldments under Axial and Bending Load: *geometrical parameters and notation*





Butt Weld Stress Concentration Factors *(K.Iida and T. Uemura, ref. 11)*

t = 20mm, g = h = 3.5mm, θ = 18°, l = h_p= 23mm, r = 0.8 mm

Pure Tension *(K.Iida and T. Uemura, ref. 11)*

$$K_t^t = 1 + \frac{1-\exp\left(-0.9\theta\sqrt{\frac{W}{2h}}\right)}{1-\exp\left(-0.45\pi\sqrt{\frac{W}{2h}}\right)} \times 2\left[\frac{1}{2.8\left(\frac{W}{t}\right)-2} \times \frac{h}{r}\right]^{0.65} \quad ;$$

$$\boldsymbol{K_t^t = 2.14}$$

$$W = t + 2h + 0.6h_p$$

Pure Bending

$$K_t^b = 1 + \frac{1-\exp\left(-0.9\theta\sqrt{\frac{W}{2h}}\right)}{1-\exp\left(-0.45\pi\sqrt{\frac{W}{2h}}\right)} \times 1.5\sqrt{\tanh\left(\frac{2r}{t}\right)} \times \tanh\left[\frac{\left(\frac{2h}{t}\right)^{0.25}}{1-\frac{r}{t}}\right] \times \left[\frac{0.13+0.65\left(1-\frac{r}{t}\right)^{4}}{\left(\frac{r}{t}\right)^{\frac{1}{3}}}\right]$$

$$W = t + 2h + 0.6h_p$$

$$\boldsymbol{K_t^b = 1.28}$$

Tensile nominal stress

$$\Delta S_t = \Delta S_t$$

Nominal bending stress

$$\Delta S_b = \frac{3e}{t}\Delta S_t = \frac{3*3}{20}\Delta S_t = 0.45\Delta S_t$$

Resultant hot spot stress

$$\Delta S_{hs} = \Delta S_t + \Delta S_b = \Delta S_t + 0.45\Delta S_t = 1.45\Delta S_t$$

Hot spot stress history

$$S_{hs,0} = 0,\ S_{hs,1} = 116,\ S_{hs,2} = 0,\ S_{hs,3} = 116, ...$$

Hot spot stress concentration factor

$$K_{t,hs}\Delta S_{hs} = K_t^b \cdot \Delta S_b + K_t^t \cdot \Delta S_t$$

$$= K_t^b \cdot 0.45\Delta S_t + K_t^t \cdot \Delta S_t$$

$$= \Delta S_t\left(0.45K_t^b + K_t^t\right);$$

$$K_{t,hs} = \frac{\Delta S_t\left(0.45K_t^b + K_t^t\right)}{\Delta S_{hs}} = \frac{\Delta S_t\left(0.45K_t^b + K_t^t\right)}{1.45\Delta S_t}$$

$$= \frac{\left(0.45K_t^b + K_t^t\right)}{1.45} = \frac{\left(0.45 \cdot 1.29 + 2.14\right)}{1.45} = 1.87$$

1st reversal

$\sigma_0 = 0$, $\varepsilon_0 = 0$,

$$\begin{cases} \dfrac{\left(1.87 \cdot 116\right)^2}{190000} = \Delta\sigma_1 * \Delta\varepsilon_1 \\ \Delta\varepsilon_1 = \dfrac{\Delta\sigma_1}{190000} + \left(\dfrac{\Delta\sigma_1}{1097}\right)^{\frac{1}{0.249}} \end{cases}$$

$\Delta\sigma_1$ = 179.87 MPa; $\Delta\varepsilon_1$ = 0.0016492;

$\sigma_1 = \sigma_0 + \Delta\sigma_1$ =0 + 179.87 = 179.87 MPa;

$\varepsilon_1 = \varepsilon_0 + \Delta\varepsilon_1$ = 0 + 0.0016492 = 0.0016492

2nd reversal

σ_1 = 179.87, ε_1 = 0.0016492

$$\begin{cases} \dfrac{\left(1.87 * 116\right)^2}{190000} = \Delta\sigma_2 * \Delta\varepsilon_2 \\ \dfrac{\Delta\varepsilon_2}{2} = \dfrac{\Delta\sigma_2}{2 * 190000} + \left(\dfrac{\Delta\sigma_2}{2 * 1097}\right)^{\frac{1}{0.249}} \end{cases}$$

$\Delta\sigma_2$ = 207.55 MPa; $\Delta\varepsilon_2$ = 0.0012468;

$\sigma_2 = \sigma_1 - \Delta\sigma_2$ = 179.87 – 207.55 = -27.68 MPa;

$\varepsilon_2 = \varepsilon_1 - \Delta\varepsilon_2$ = 0.0016492 - 0.0012468= 0.000402

$$\sigma_{m2} = \frac{\sigma_1 + \sigma_2}{2} = \frac{179.87 + (-27.68)}{2} = 76.09MPa$$

$$\frac{\Delta\varepsilon_2}{2} = \frac{\sigma'_f - \sigma_{m2}}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c$$

$$\frac{0.0012468}{2} = \frac{1014 - 76.09}{190000}\left(2N_f\right)^{-0.132} + 0.271\left(2N_f\right)^{-0.451}$$

$$N_f = 15.095 \times 10^6 \quad cycles$$

(end of butt weld)

Fillet Weld Stress Concentration Factors *(K.Iida and T. Uemura, ref. 11)*

t = 20mm, g = h = 3.5mm, θ = 18º, l = h_p= 23mm, r = 0.8 mm

Pure Tension

$$K_t^t = 1 + \frac{1-\exp\left(-0.9\theta\sqrt{\frac{W}{2h}}\right)}{1-\exp\left(-0.45\pi\sqrt{\frac{W}{2h}}\right)} \times \left[\frac{1}{2.8\left(\frac{W}{t}\right)-2} \times \frac{h}{r}\right]^{0.65};$$

$$\boldsymbol{K_t^t} = 2.42$$

$$W = (t+2h) + 0.3(t_p + 2h_p)$$

Pure Bending

$$K_t^b = 1 + \frac{1-\exp\left(-0.9\theta\sqrt{\frac{W}{2h}}\right)}{1-\exp\left(-0.45\pi\sqrt{\frac{W}{2h}}\right)} \times 1.9\sqrt{\tanh\left(\frac{2t_p}{t+2h}+\frac{2r}{t}\right)} \times \tanh\left[\frac{\left(\frac{2h}{t}\right)^{0.25}}{1-\frac{r}{t}}\right] \times \left[\frac{0.13+0.65\left(1-\frac{r}{t}\right)^4}{\left(\frac{r}{t}\right)^{\frac{1}{3}}}\right]$$

$$W = (t+2h) + 0.3(t_p + 2h_p)$$

$$\boldsymbol{K_t^b} = 2.85$$

Tensile nominal stress

$$\Delta S_t = \Delta S_t$$

Nominal bending stress

$$\Delta S_b = \frac{3e}{2t}\Delta S_t = \frac{3*3}{2\cdot 20}\Delta S_t = 0.225\Delta S_t$$

Resultant hot spot stress

$$\Delta S_{hs} = \Delta S_t + \Delta S_b = \Delta S_t + 0.225\Delta S_t = 1.225\Delta S_t$$

Hot spot stress history

$$S_{hs,0} = 0,\ S_{hs,1} = 98,\ S_{hs,2} = 0,\ S_{hs,3} = 98, \ldots$$

Hot spot stress concentration factor

$$K_{t,hs}\Delta S_{hs} = K_t^b \cdot \Delta S_b + K_t^t \cdot \Delta S_t$$

$$= K_t^b \cdot 0.225\Delta S_t + K_t^t \cdot \Delta S_t$$

$$= \Delta S_t\left(0.225K_t^b + K_t^t\right);$$

$$K_{t,hs} = \frac{\Delta S_t\left(0.225K_t^b + K_t^t\right)}{\Delta S_{hs}} = \frac{\Delta S_t\left(0.225K_t^b + K_t^t\right)}{1.225\Delta S_t}$$

$$= \frac{\left(0.225K_t^b + K_t^t\right)}{1.225} = \frac{\left(0.225\cdot 2.85 + 2.42\right)}{1.225} = 2.5$$

1st reversal

$\sigma_0 = 0$, $\varepsilon_0 = 0$,

$$\begin{cases} \dfrac{\left(2.5\cdot 98\right)^2}{190000} = \Delta\sigma_1 * \Delta\varepsilon_1 \\ \Delta\varepsilon_1 = \dfrac{\Delta\sigma_1}{190000} + \left(\dfrac{\Delta\sigma_1}{1097}\right)^{\frac{1}{0.249}} \end{cases}$$

$\Delta\sigma_1 = 189.57$ MPa; $\Delta\varepsilon_1 = 0.001865$;

$\sigma_1 = \sigma_0 + \Delta\sigma_1 = 0 + 189.57 = 189.57$ MPa;

$\varepsilon_1 = \varepsilon_0 + \Delta\varepsilon_1 = 0 + 0.001865 = 0.001865$

2nd reversal

$\sigma_1 = 189.57$, $\varepsilon_1 = 0.001865$

$$\begin{cases} \dfrac{\left(2.5\cdot 98\right)^2}{190000} = \Delta\sigma_2 * \Delta\varepsilon_2 \\ \dfrac{\Delta\varepsilon_2}{2} = \dfrac{\Delta\sigma_2}{2*190000} + \left(\dfrac{\Delta\sigma_2}{2*1097}\right)^{\frac{1}{0.249}} \end{cases}$$

$\Delta\sigma_2 = 230.05$ MPa; $\Delta\varepsilon_2 = 0.001444$;

$\sigma_2 = \sigma_1 - \Delta\sigma_2 = 189.57 – 230.5 = -40.93$ MPa;

$\varepsilon_2 = \varepsilon_1 - \Delta\varepsilon_2 = 0.001865 - 0.001444 = 0.000421$

$$\sigma_{m2} = \frac{\sigma_1 + \sigma_2}{2} = \frac{189.57 + (-40.93)}{2} = 74.32 MPa$$

$$\frac{\Delta\varepsilon_2}{2} = \frac{\sigma'_f - \sigma_{m2}}{E}(2N_f)^b + \varepsilon'_f (2N_f)^c$$

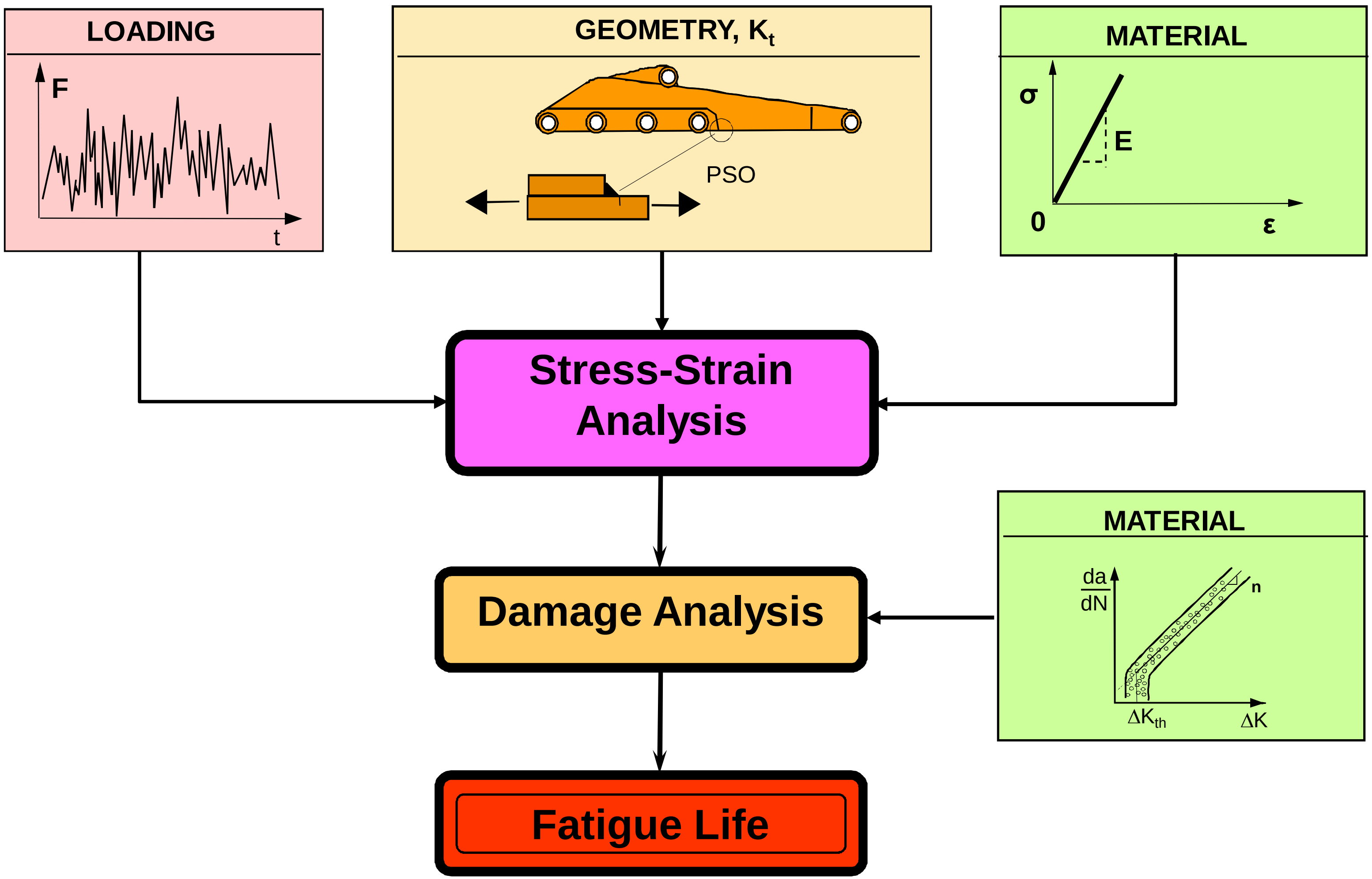
$$\frac{0.001444}{2} = \frac{1014 - 74.32}{190000}(2N_f)^{-0.132} + 0.271(2N_f)^{-0.451}$$

$$N_f = 7.1517 \times 10^6 \quad cycles$$

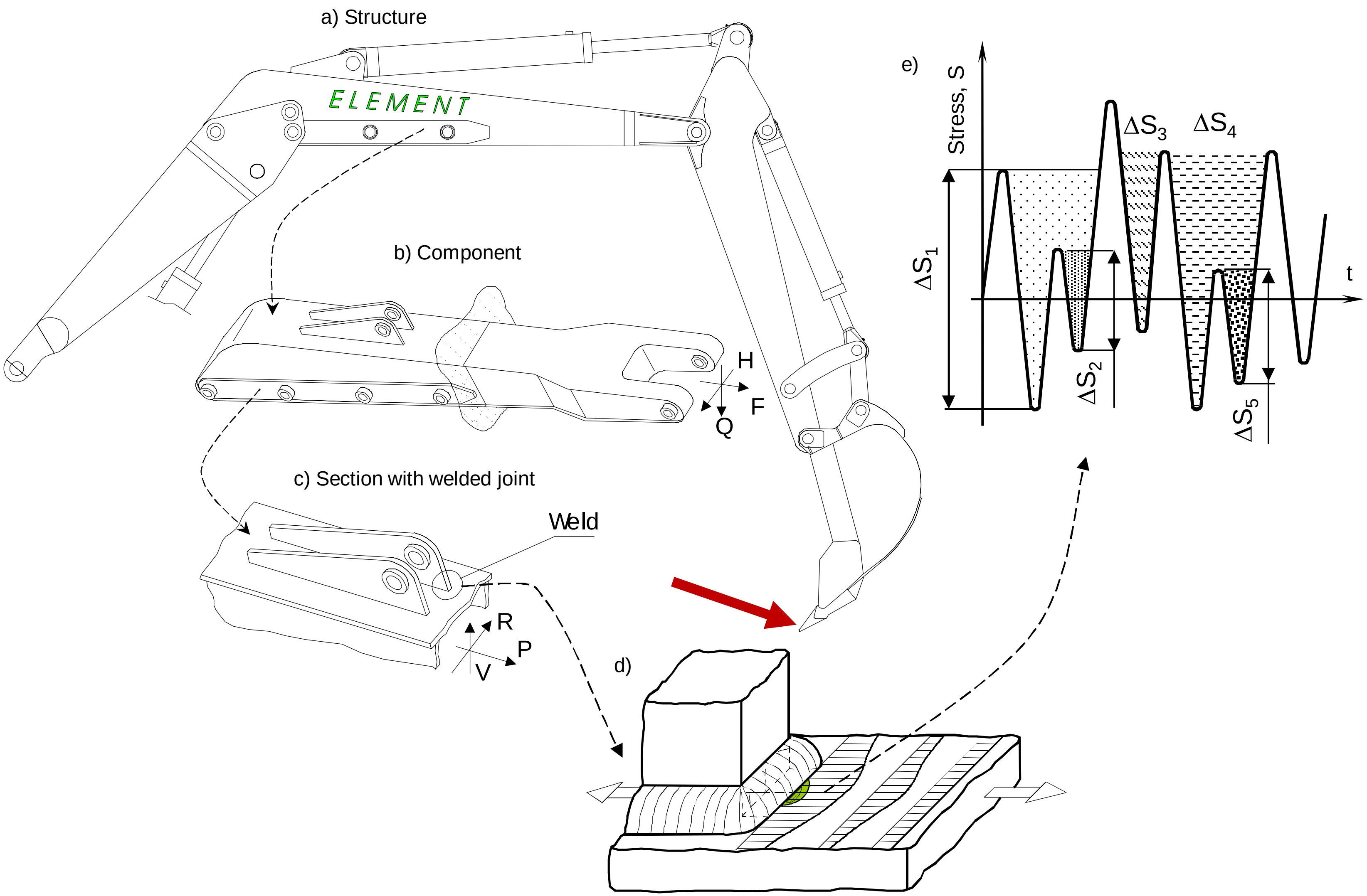
(end of fillet weld)

Fracture Mechanics Approach to Fatigue Analysis of Weldments (da/dN-ΔK)

Information path for fatigue life estimation based on the da/dN-ΔK method

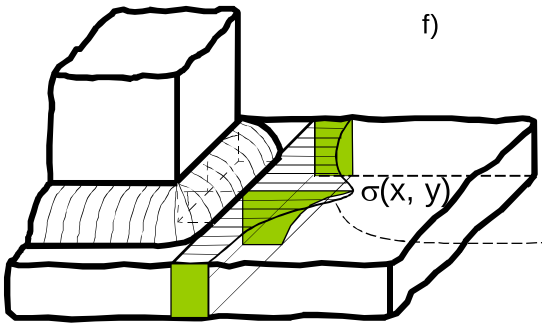


Steps in the Fatigue Life Prediction Procedure Based on the da/dN-ΔK Approach



Steps in Fatigue Life Prediction Procedure Based on the da/dN-ΔK Approach *(cont'd)*

f)



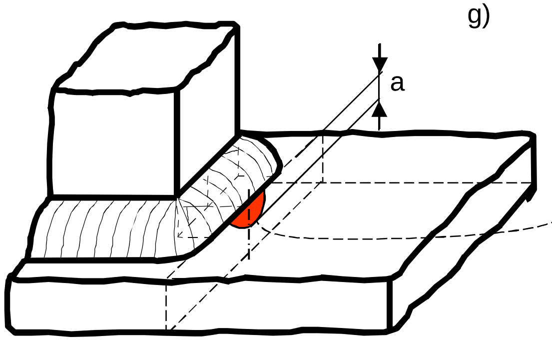
Stress intensity factor, K (indirect method)

Weight function, m(x,y)

$$K = \iint_A \sigma(x,y)\, m(x,y)\, dxdy$$

$$Y = \frac{K}{\sigma_n \sqrt{\pi a}}$$

g)



Stress intensity factor, K (direct method)

$$K_I = \sigma_{yFE}\sqrt{2\pi x_{FE}}$$

or

$$K = \sqrt{E\frac{dU}{da}} = \sqrt{EG}$$

$$Y = \frac{K}{\sigma_n \sqrt{\pi a}}$$

h)

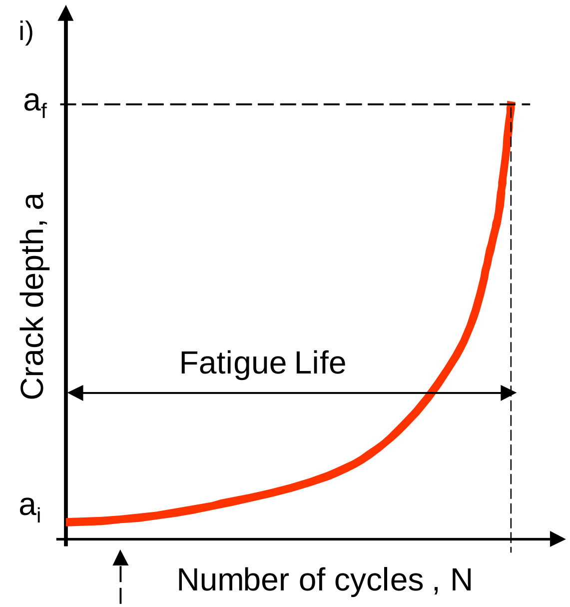
Integration of Paris' equation

$$\Delta a_i = C\left(\Delta K_i\right)^m \Delta N_i$$

$$a_f = a_0 + \sum_{i=1}^{N} \Delta a_i$$

$$N = \sum \Delta N_i$$

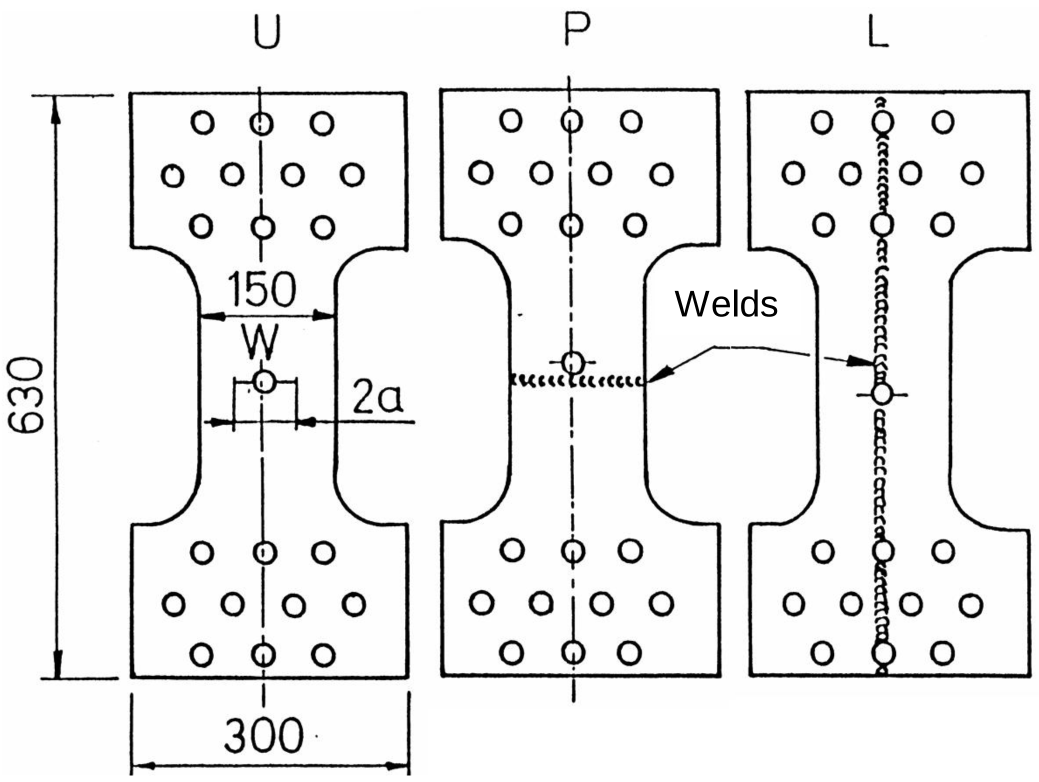
i)



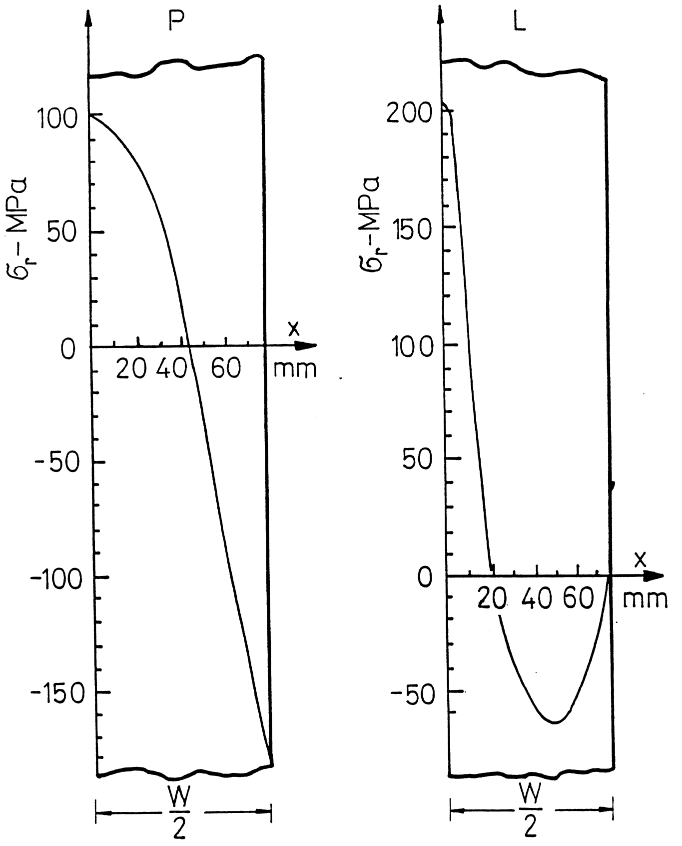
Material properties used in fatigue analyses of weldments by the Fracture Mechanics Method (da/dN-ΔK)

Residual Stress Distributions in Welded Joints; Butt Weldments

Specimens

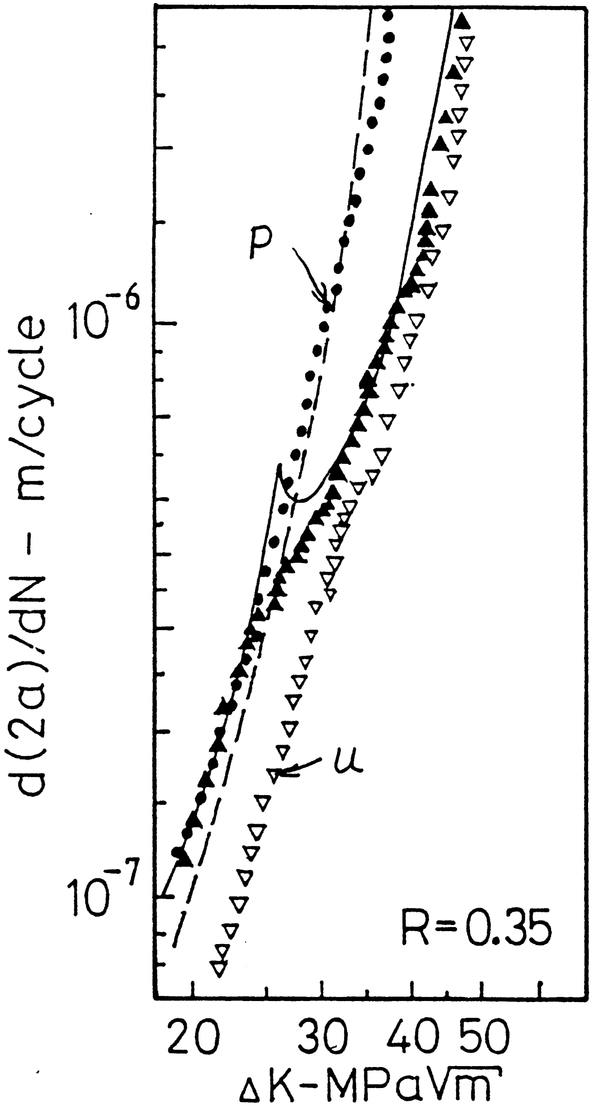


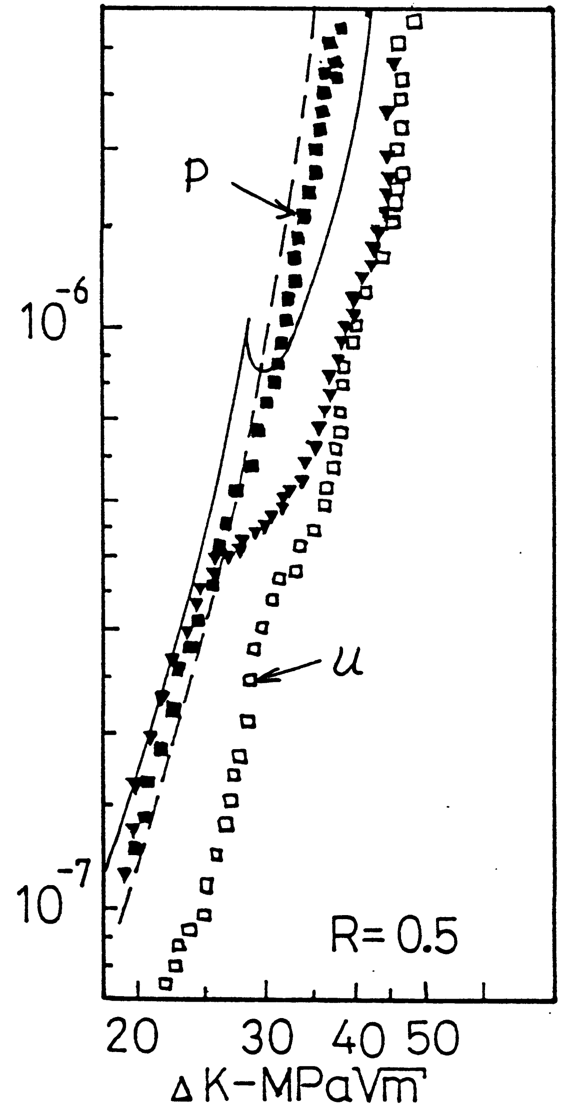
Residual stress distributions



Residual Stress Effect on the Fatigue Crack Growth Rate

(Specimens P and L and U)





Fatigue cracks in weldments may grow through the HAZ but most often they initiate in the HAZ but grow away from the weld and through the base metal.

Therefore the Base Metal fatigue crack growth properties are used for fatigue crack growth analysis of weldments.

$$\frac{da}{dN} = C\left(\Delta K\right)^{m}$$

Where: C and m - Base Metal properties

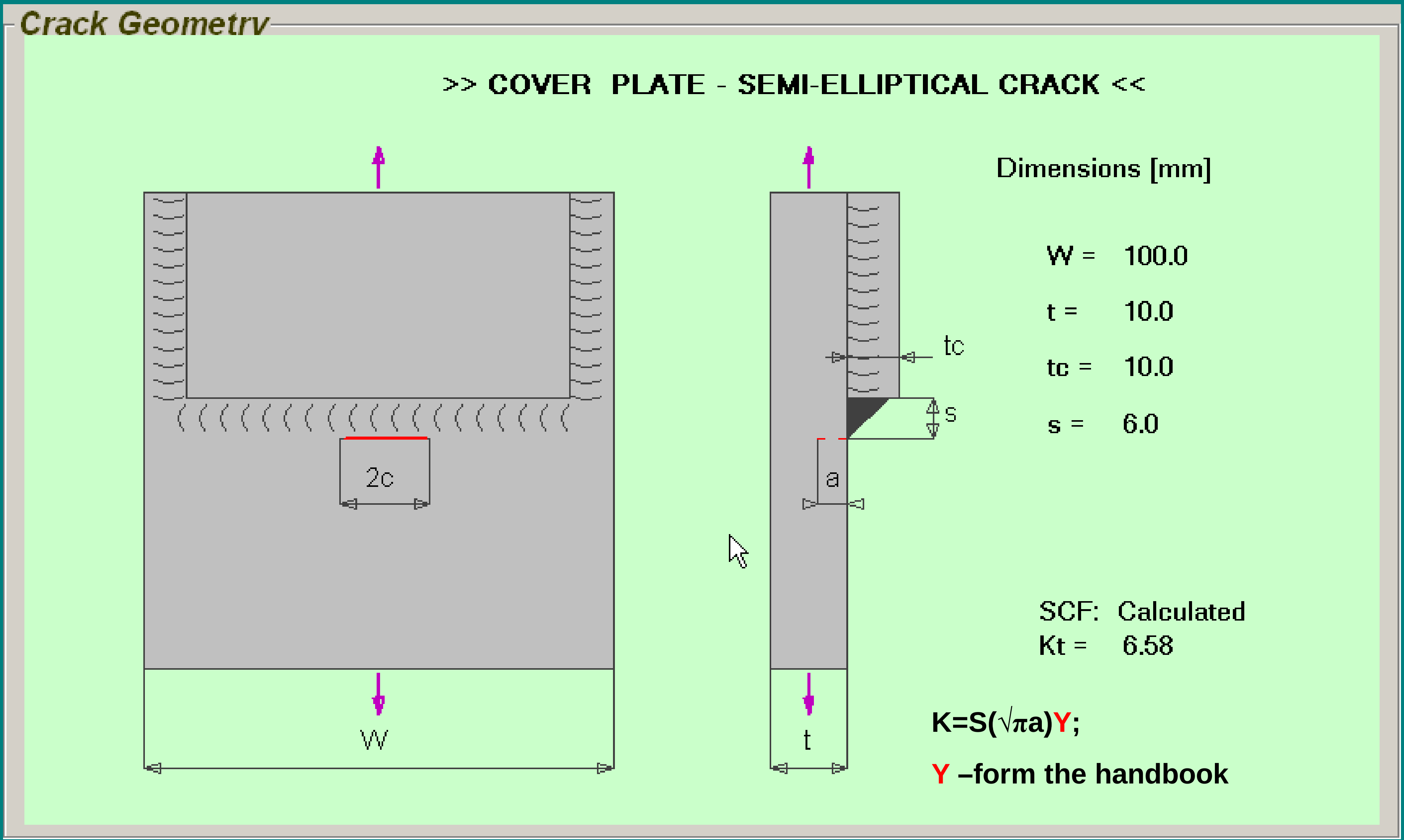
Calculation of Stress Intensity Factors (K) for Cracks in Weldments

There are two methods available for obtaining stress intensity factors for cracks in weldments:

a) **The Handbook ready made stress intensity factors K for cracks in weldments like the handbook of SIFs by Y. Murakami et. al, (editor), Stress Intensity Factors Handbook, Pergamon Press, Oxford, 1987 (unfortunately the number of solutions for cracks in weldemnts is very limited);** ***The ready made K solutions are usually obtained for fixed geometry (such as specific geometrical dimensions of the weld) and they can't be used for estmating the K factor resultin from residual stresses.***

b) **The Weight Function (WF) method; The WFs make it possible to solve a wide variety of K problems for cracks in weldments by using a very limited number of general weight functions. The same WF can be used for the estimation of K factors associated with the presence of residual stresses.**

Handbook SIF

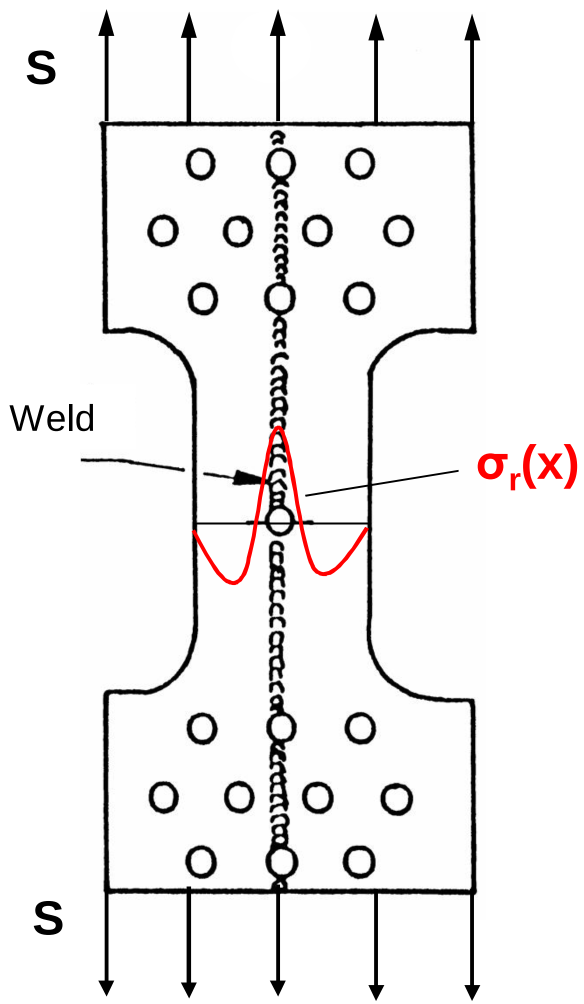


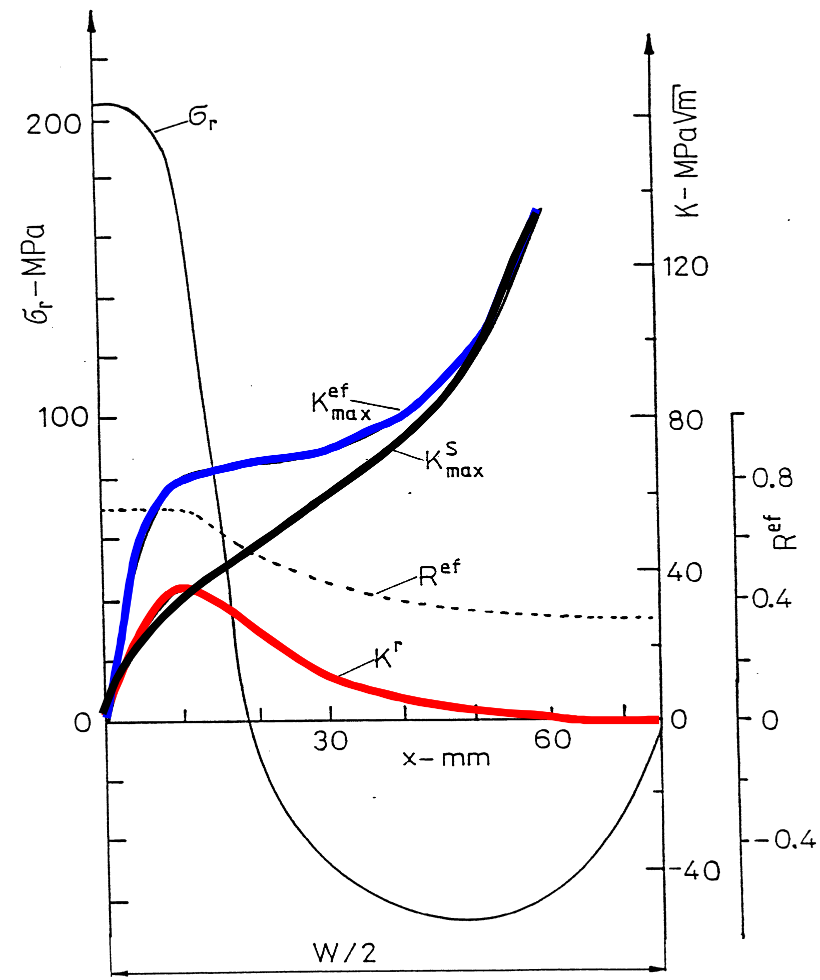
K=S($\sqrt{\pi a}$)Y;

Y –form the handbook

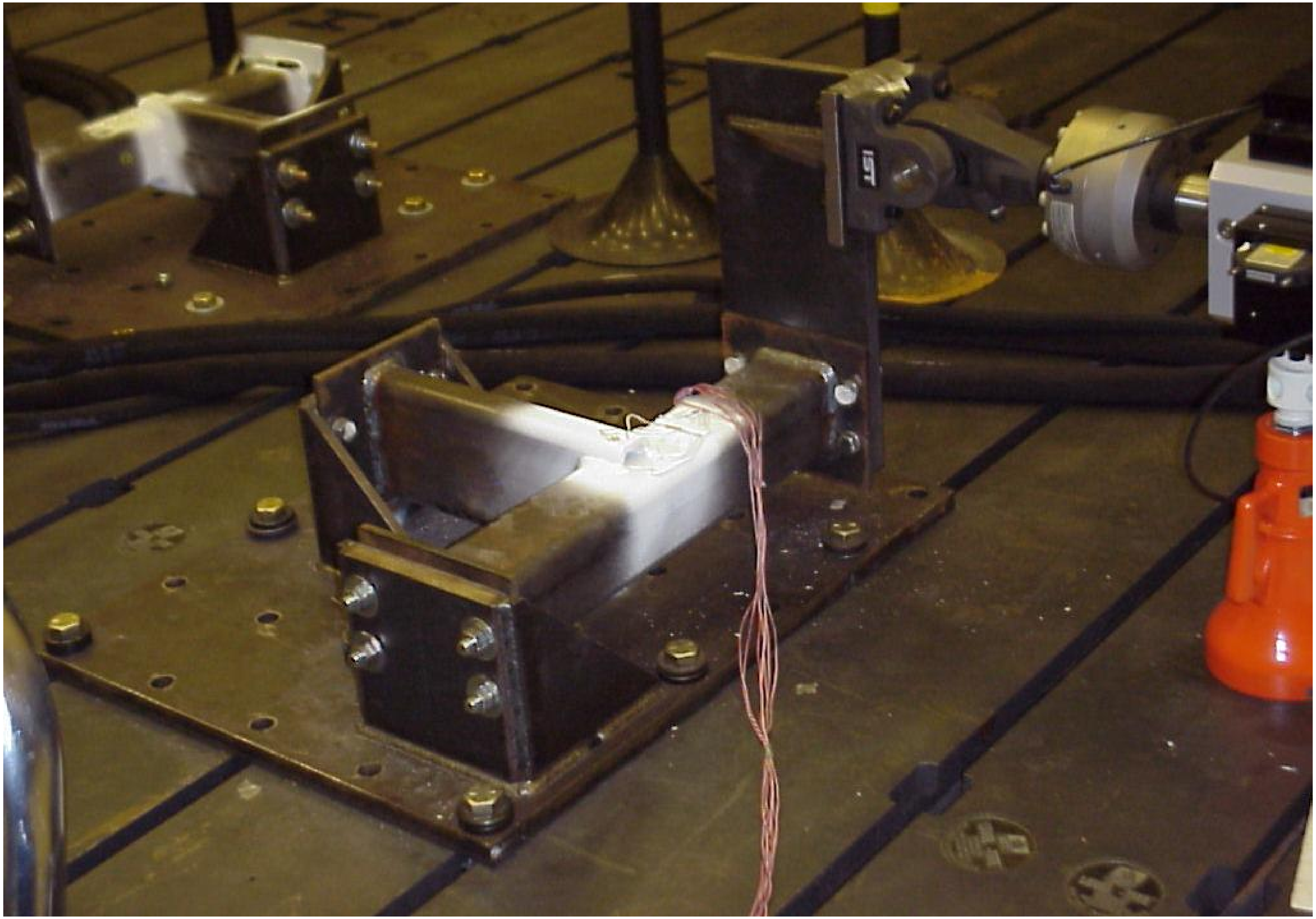
The Effective and the Residual Stress Intensity Factors in a Butt Weldment

$$K_S = S\sqrt{\pi a}\cdot Y \qquad \textit{and} \qquad K_r = \int_0^a \sigma_r(x)\cdot m(x,a)dx$$



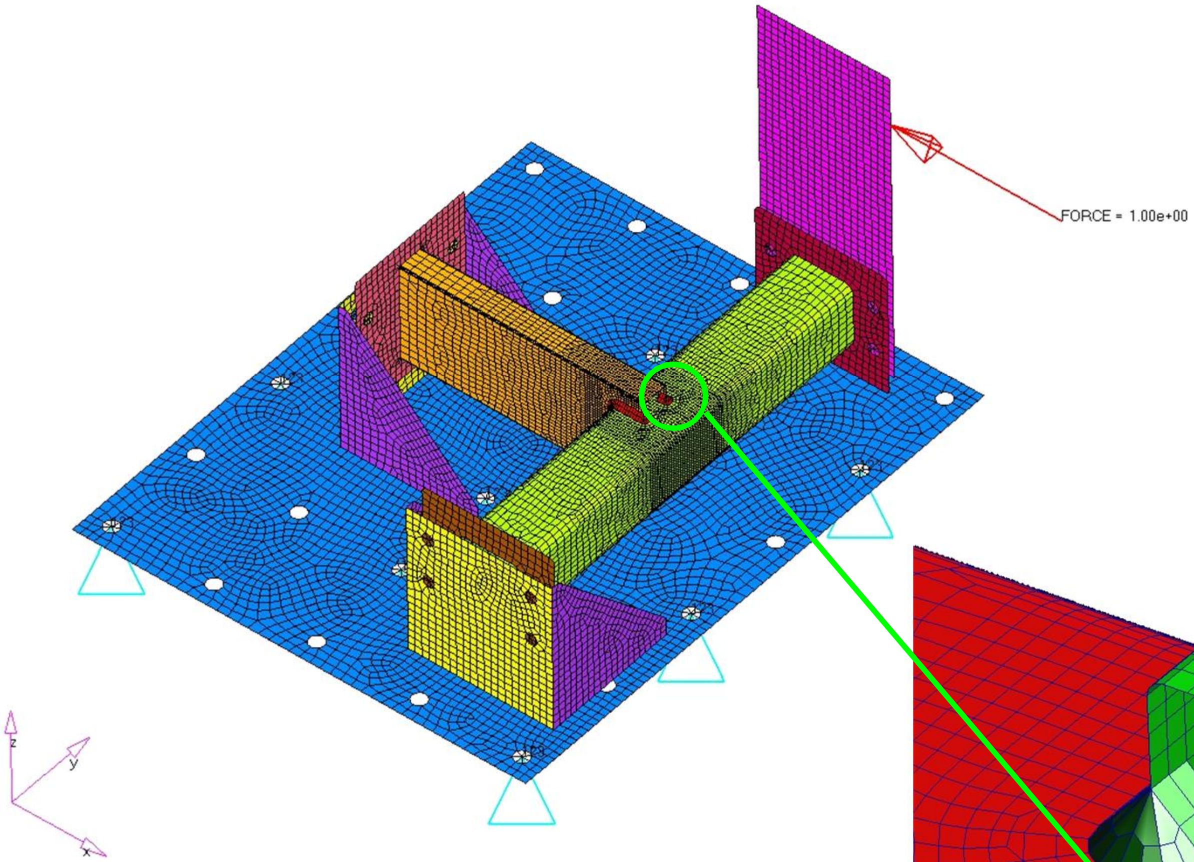


Tubular Welded Joint under Torsion and Bending

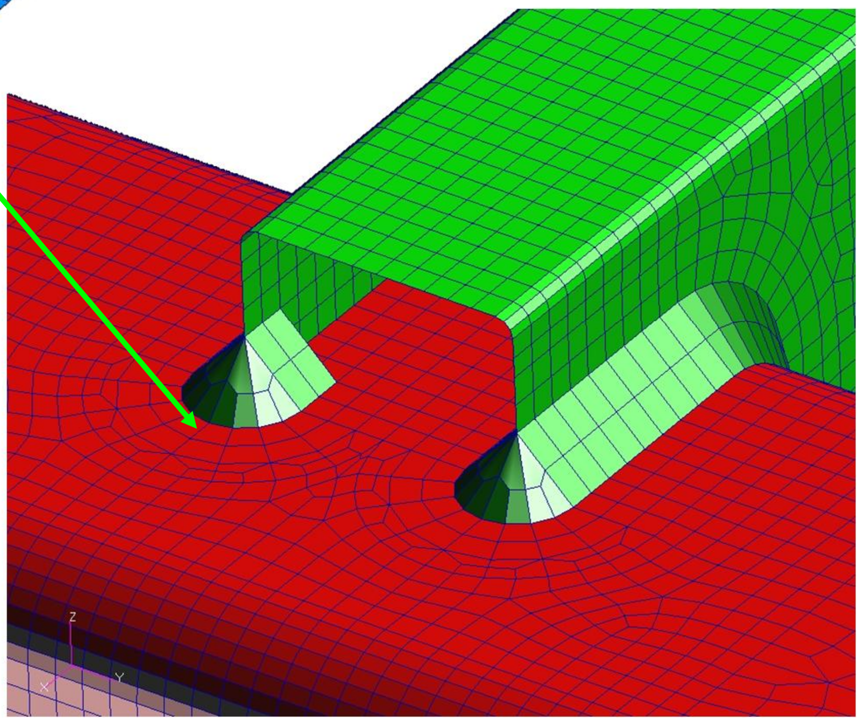


Courtesy of John Deere Co.

Shell Element Model Details

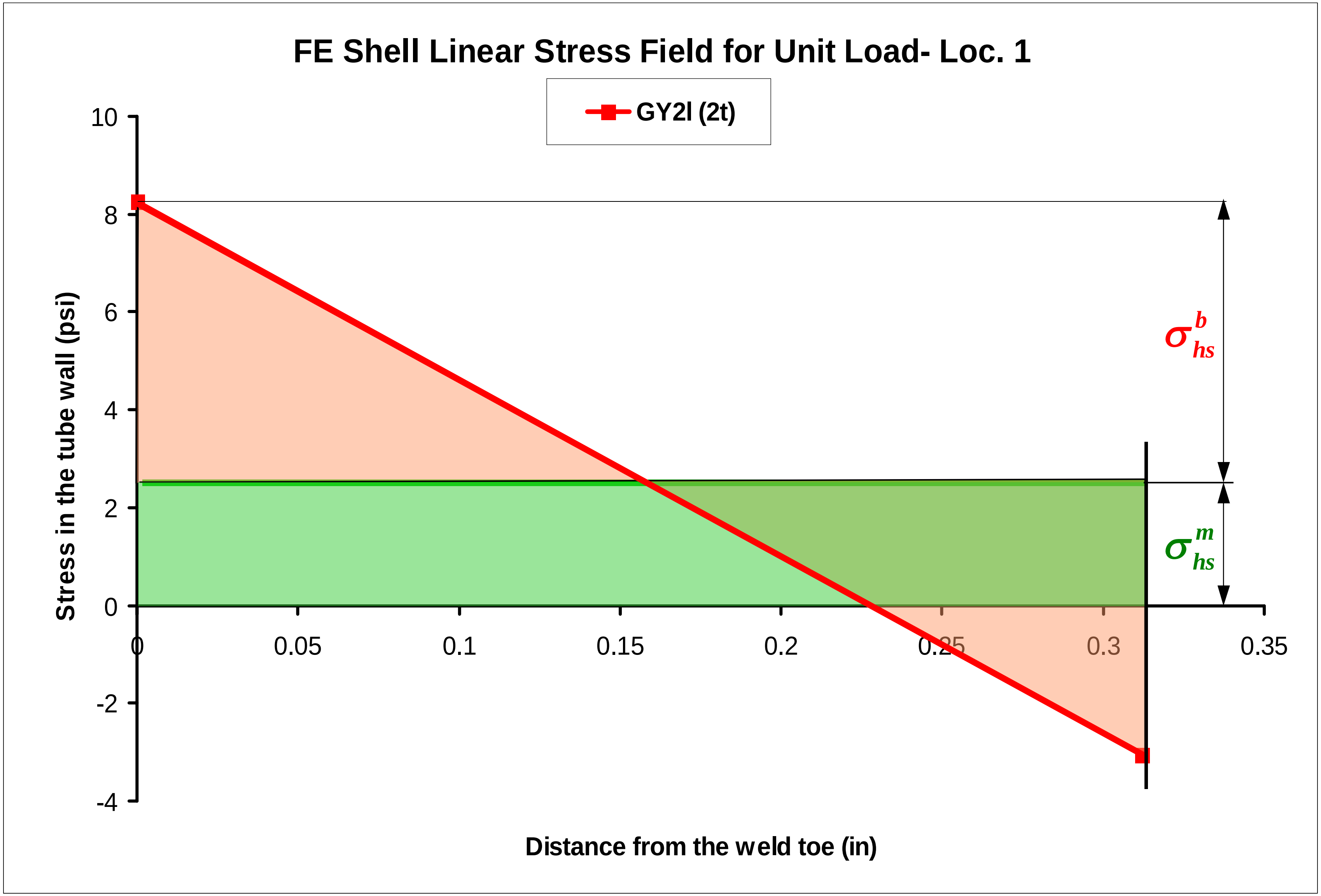


19197 nodes
18858 elements (linear quads)
114069 dof
Follows GY-2 modeling practice



Material:
A22H Steel (ASTM A500 Cold Formed Steel for Structural Tubing)

Courtesy of John Deere Co.



FE Shell Linear Stress Field for Unit Load- Loc. 1
GY2l (2t)
Stress in the tube wall (psi)
10
8
6
4
2
0
-2
-4
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
σ_{hs}^{b}
σ_{hs}^{m}
Distance from the weld toe (in)

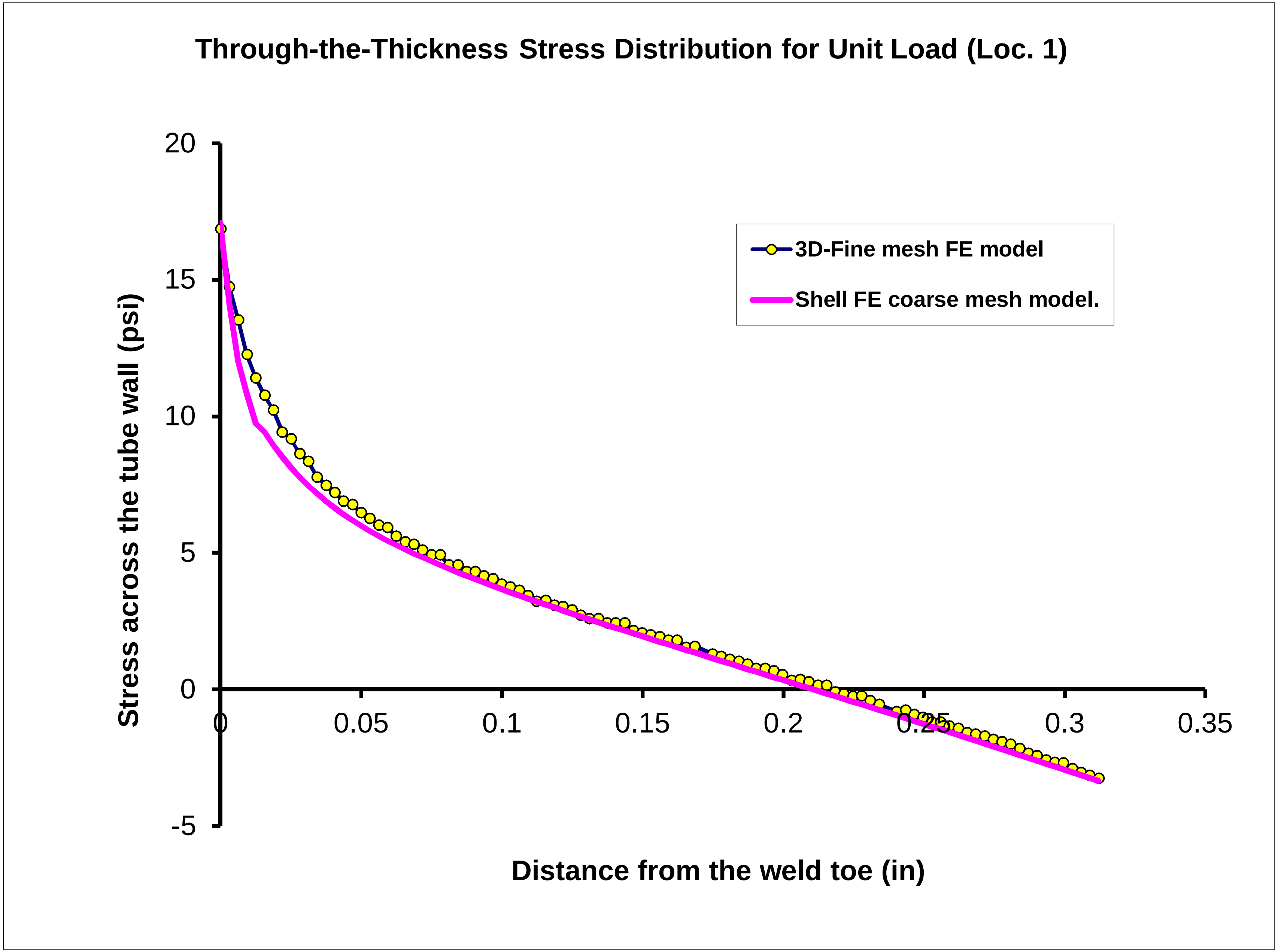
Theoretical through-thickness stress distribution

(Monahan's equations for mixed mode loading, i.e. simultaneous axial and bending)

$$\sigma\left(x=0,y\right)^{m}=\frac{K_t^t\sigma_{hs}^m}{2\sqrt{2}}\left[\left(\frac{y}{r}+\frac{1}{2}\right)^{-\frac{1}{2}}+\frac{1}{2}\left(\frac{y}{r}+\frac{1}{2}\right)^{-\frac{3}{2}}\right]\frac{1}{G_m}$$

$$\sigma\left(x=0,y\right)^{b}=\frac{K_t^b\sigma_{hs}^b}{2\sqrt{2}}\left[\left(\frac{y}{r}+\frac{1}{2}\right)^{-\frac{1}{2}}+\frac{1}{2}\left(\frac{y}{r}+\frac{1}{2}\right)^{-\frac{3}{2}}\right]\frac{1-2\left(\frac{y}{t}\right)}{G_b}$$

$$\sigma\left(y\right)=\sigma\left(y\right)^{m}+\sigma\left(y\right)^{b}=\left[\frac{K_t^m\sigma_{hs}^m}{2\sqrt{2}}\cdot\frac{1}{G_m}+\frac{K_t^b\sigma_{hs}^b}{2\sqrt{2}}\cdot\frac{1-2\left(\frac{y}{t}\right)}{G_b}\right]\left[\left(\frac{y}{r}+\frac{1}{2}\right)^{-\frac{1}{2}}+\frac{1}{2}\left(\frac{y}{r}+\frac{1}{2}\right)^{-\frac{3}{2}}\right]$$



Through-the-Thickness Stress Distribution for Unit Load (Loc. 1)
3D-Fine mesh FE model
Shell FE coarse mesh model.
Stress across the tube wall (psi)
Distance from the weld toe (in)
20
15
10
5
0
-5
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35

Fig.15

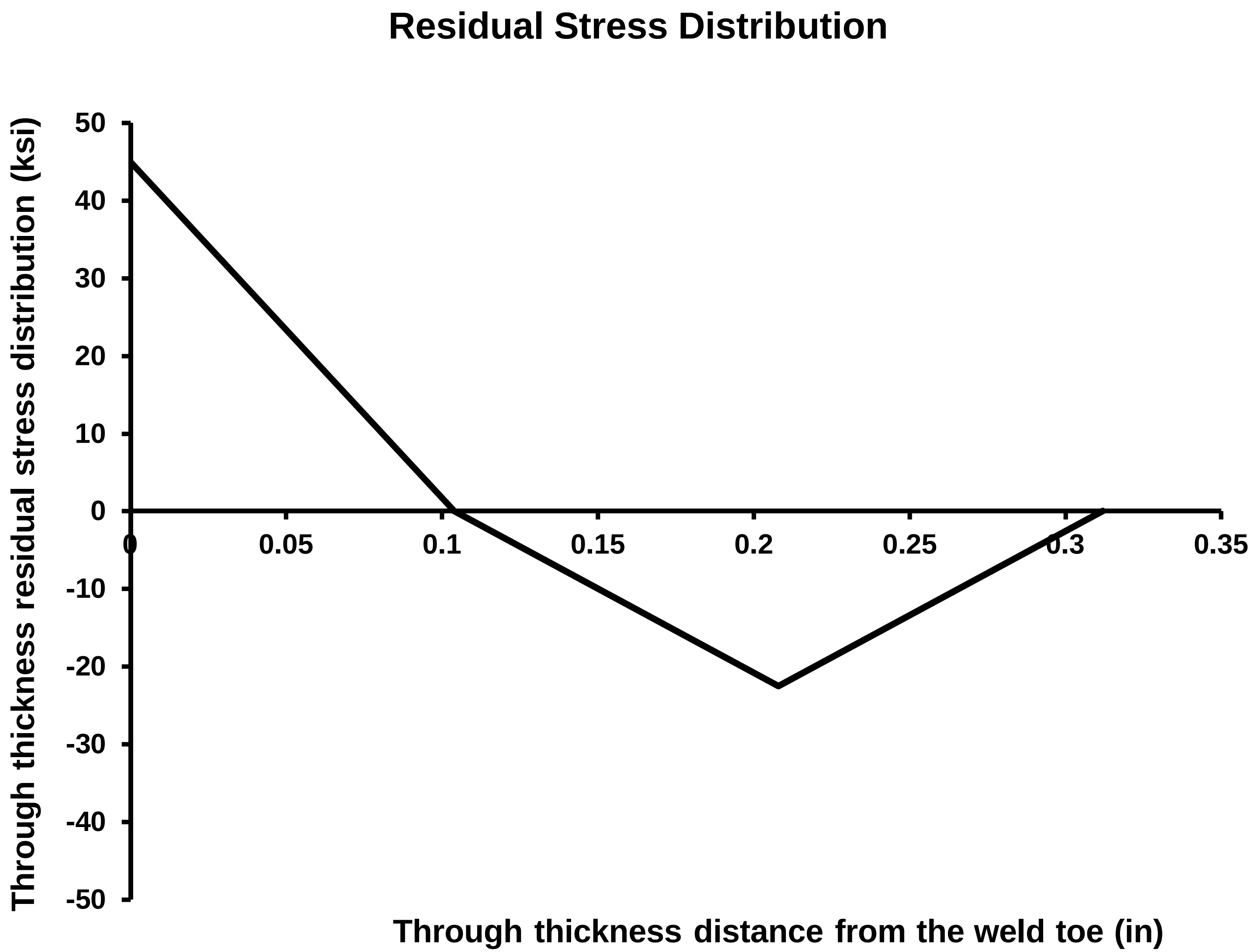
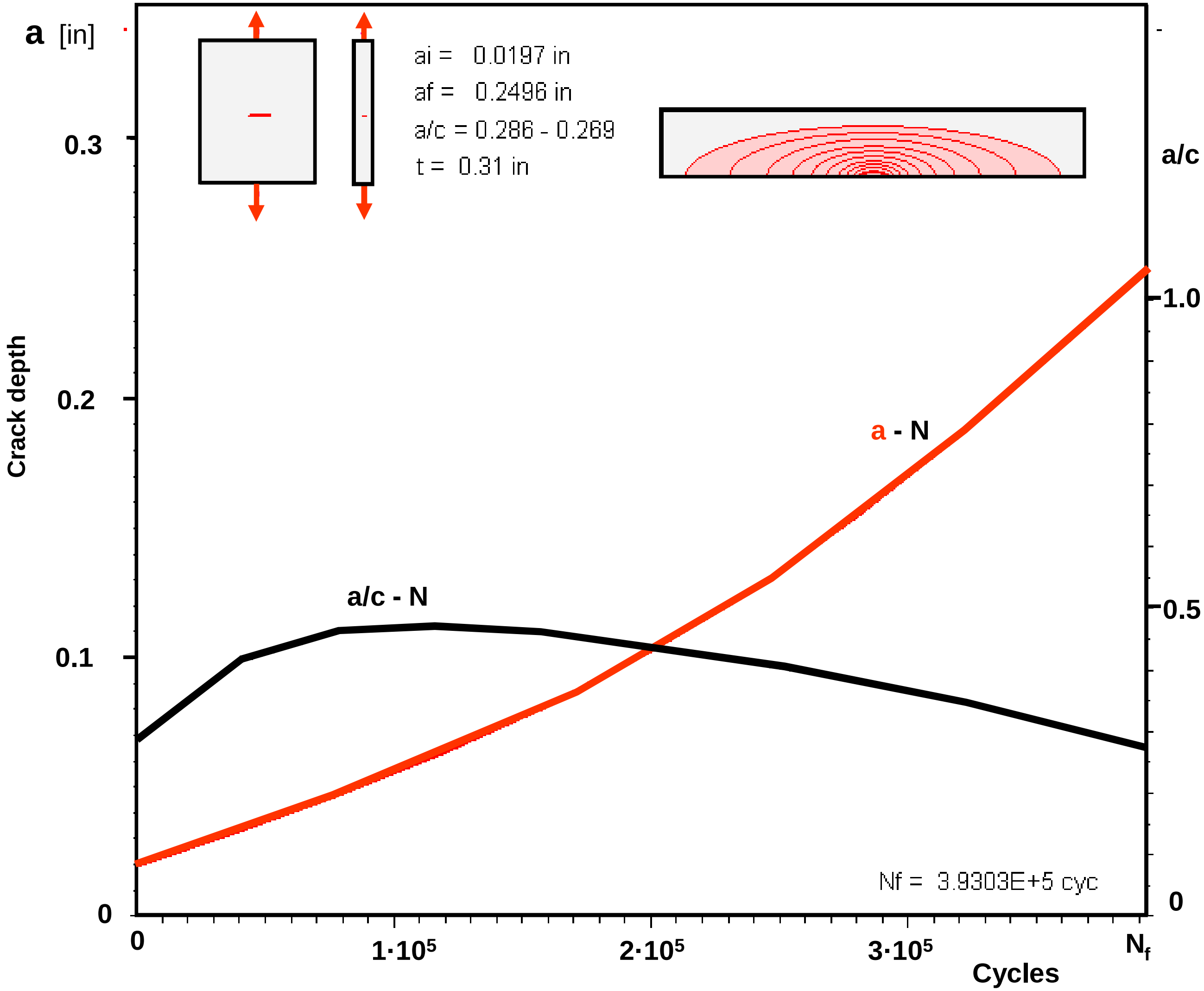
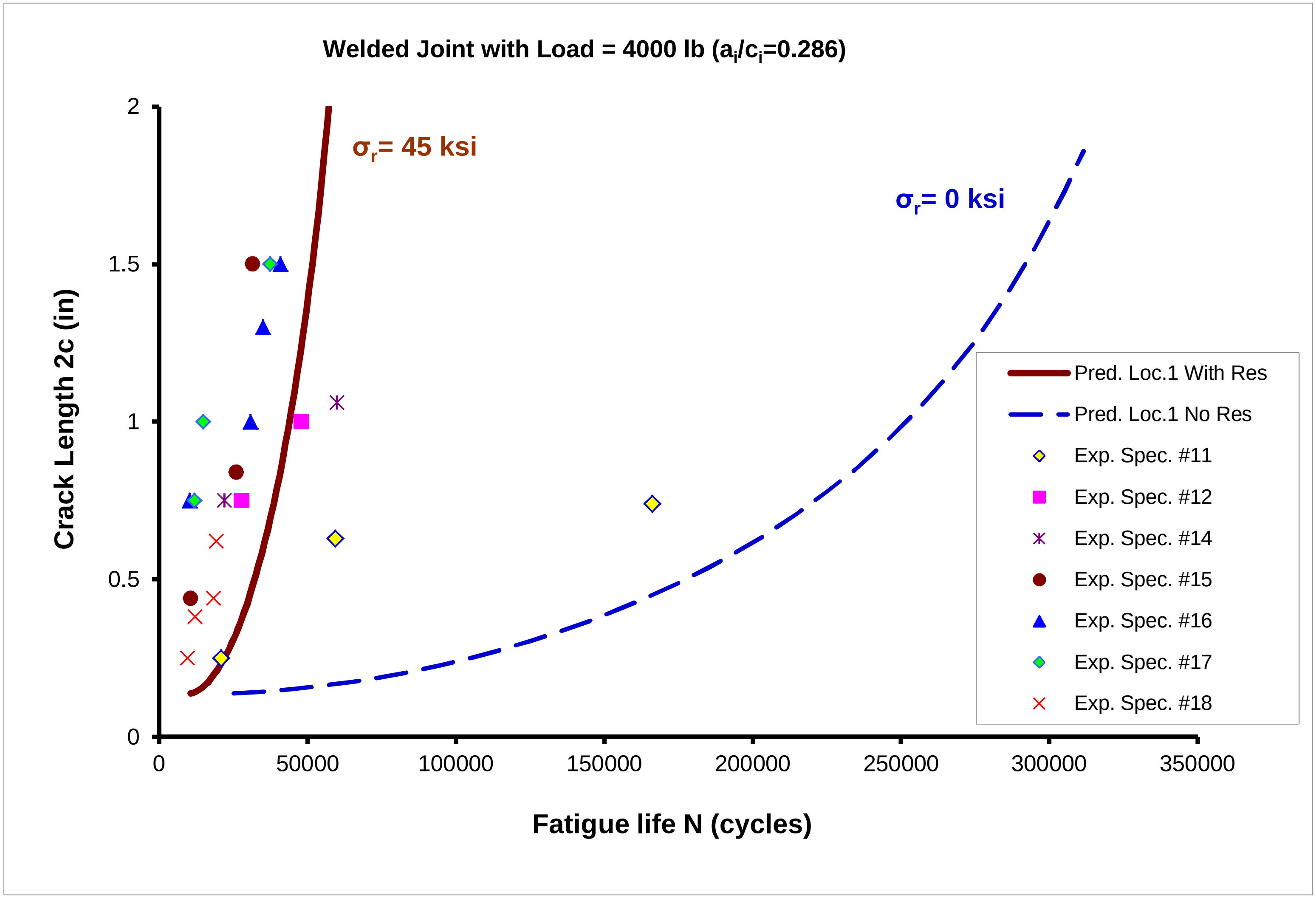

Residual Stress Distribution
Through thickness residual stress distribution (ksi)
50
40
30
20
10
0
-10
-20
-30
-40
-50
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
Through thickness distance from the weld toe (in)

Fig. 17

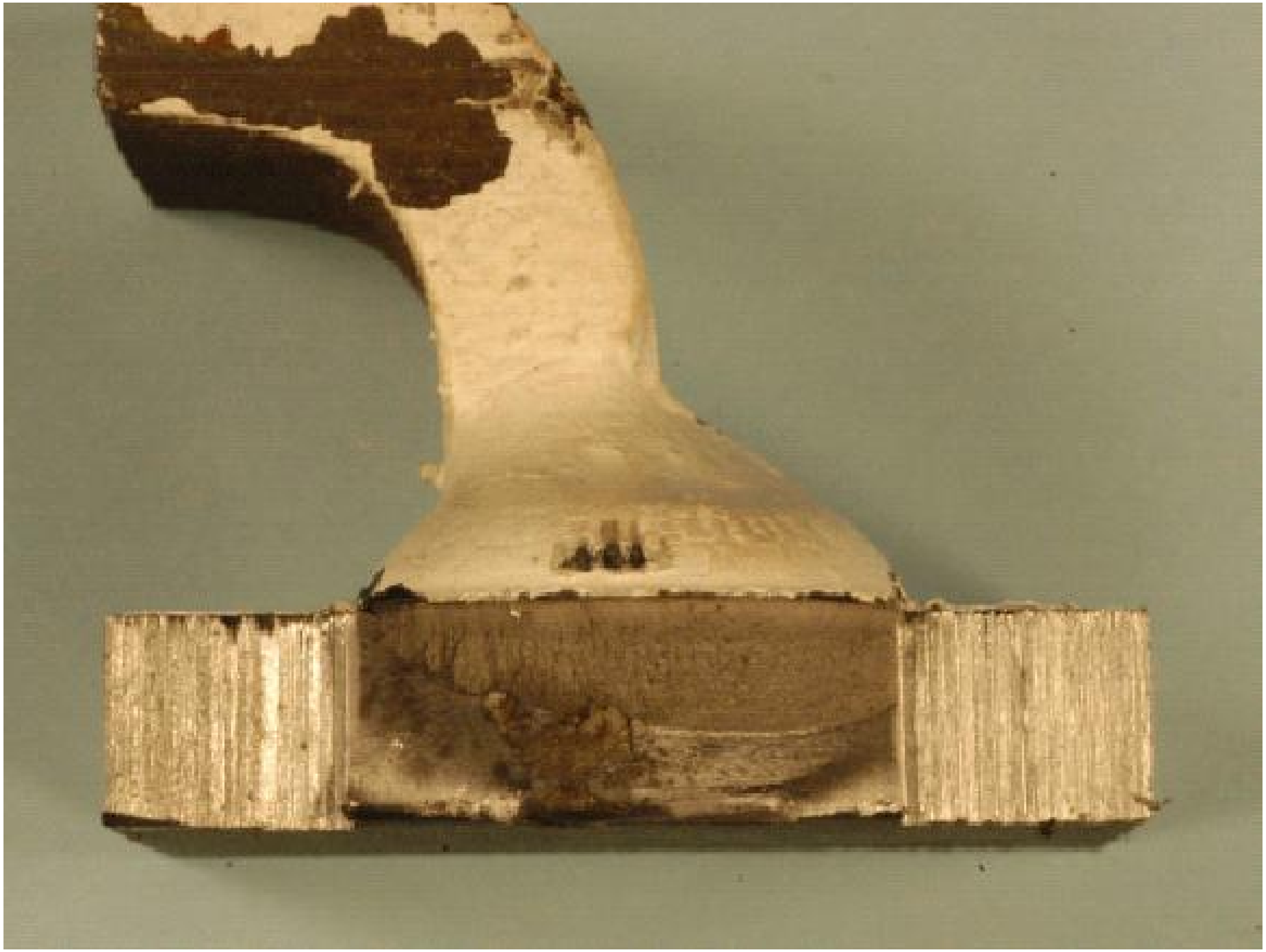
Simulated Fatigue Crack Growth and Fatigue Crack Evolution in a Weldment Based on the Non-Linear Through-the-Thickness Stress Distribution

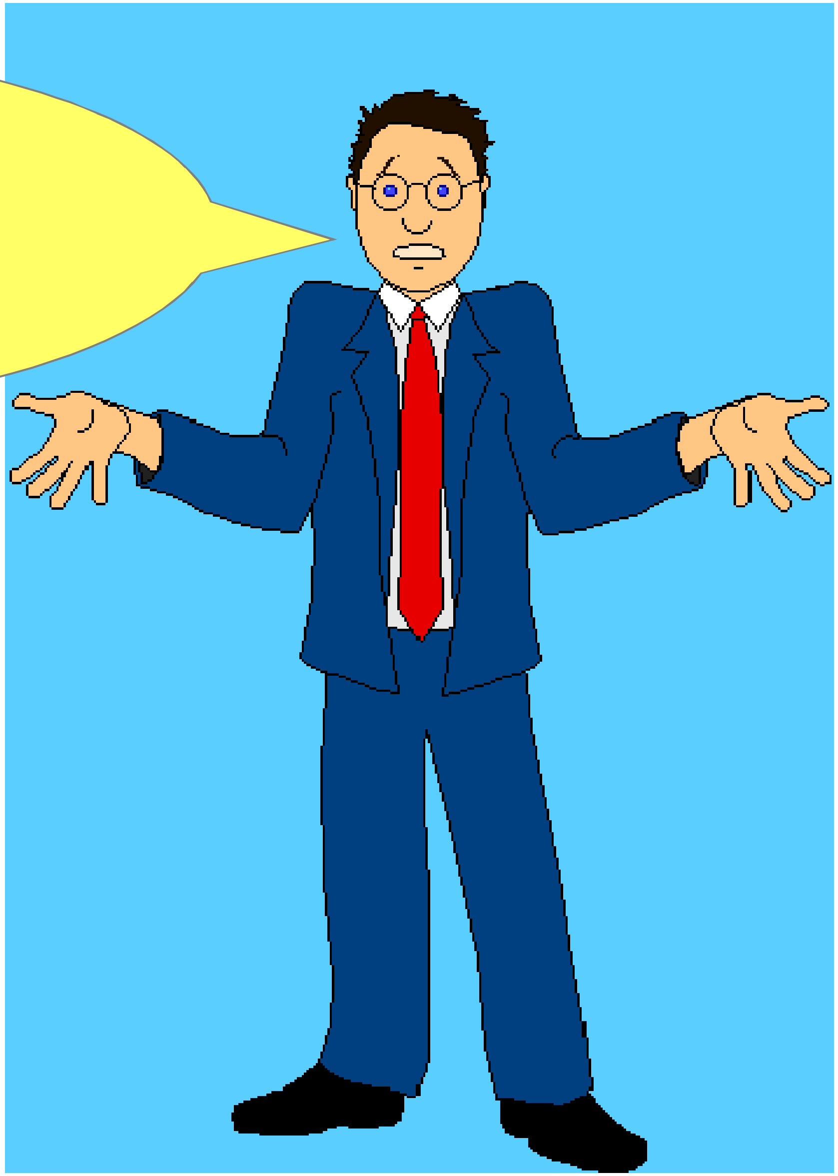


Experimental and Simulated Fatigue Crack Growth Curves (2c-N)



Geometry of the real final fatigue crack





Thank You !

