FATIGUE FROM VARIABLE AMPLITUDE LOADING

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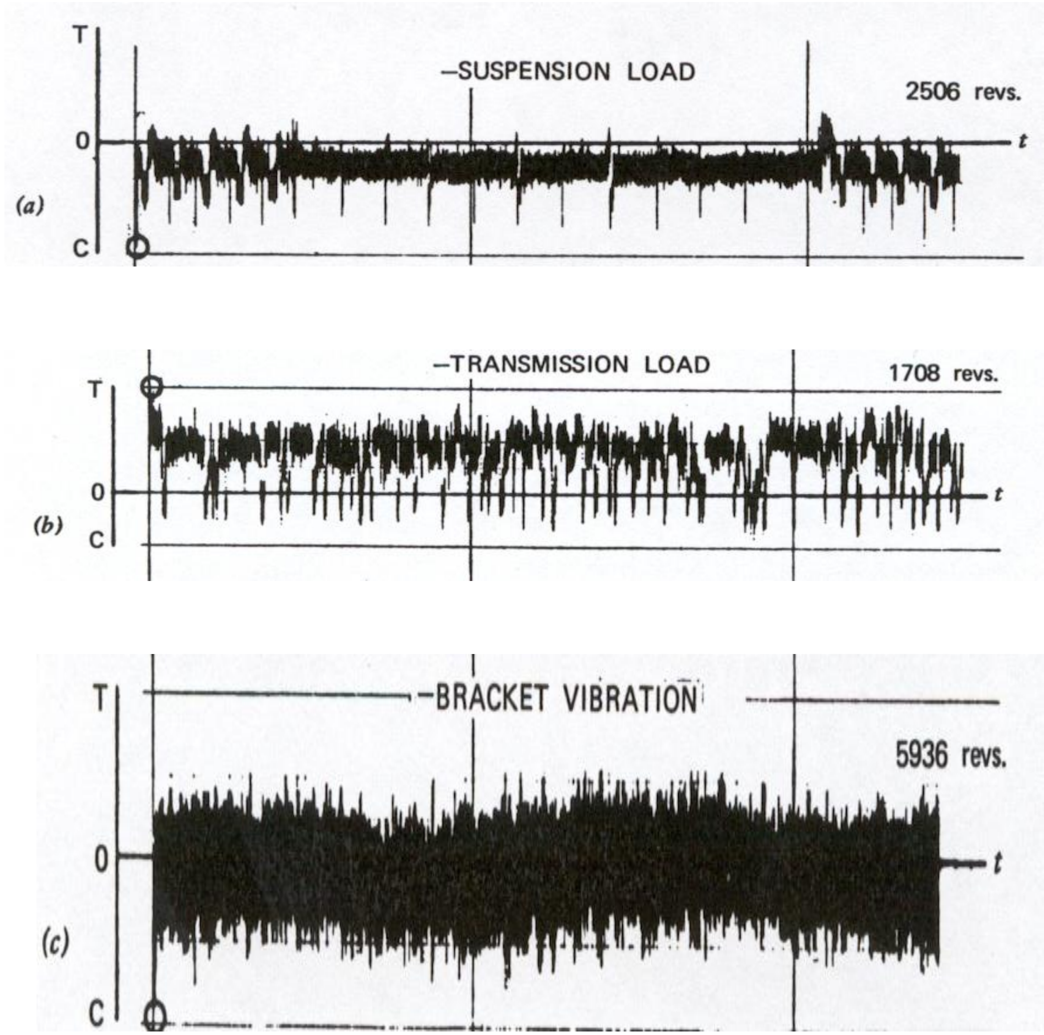
- SPECTRUM LOADS AND CUMULATIVE DAMAGE
- DAMAGE QUANTIFICATION AND THE CONCEPTS OF DAMAGE FRACTION AND ACCUMULATION
- CUMULATIVE DAMAGE THEORIES
- LOAD INTERACTION AND SEQUENCE EFFECTS
- CYCLE COUNTING METHODS
- LIFE ESTIMATION USING
  - STRESS-LIFE APPROACH
  - STRAIN-LIFE APPROACH
  - CRACK GROWTH MODELS
- SIMULATING SERVICE LOADS & DIGITAL PROTOTYPING

SPECTRUM LOADS & CUMULATIVE DAMAGE

- Service loads are usually variable amplitude
- **Realistic** representation of service loads is a key ingredient to successful fatigue analysis & design. It is important to:
  - accurately measure the applied loads on an existing component or structure
  - predict loads on a component or structure that does not yet exist.



SPECTRUM LOADS & CUMULATIVE DAMAGE

- To measure the load history, transducers (most commonly strain gages) are attached to the critical areas of the component (often identified by FEA, or experimentally).

- The acquired data from the transducers are usually recorded and stored by a computer or by other devices.

- The recorded data may be filtered to isolate the primary loads from noise.

- The recorded data are then often summarized or compressed by cycle counting methods in order to simplify the fatigue damage computations.

SPECTRUM LOADS & CUMULATIVE DAMAGE

- Real-life load histories can be applied directly to small test specimens, components, subassemblies, and even entire products with closed-loop electro-hydraulic test systems .

- Historically, complex load histories are often replaced by more **simplified loadings**, such as the block programs shown in Figs. 9.2*a* and 9.2*b*, or by constant amplitude tests.

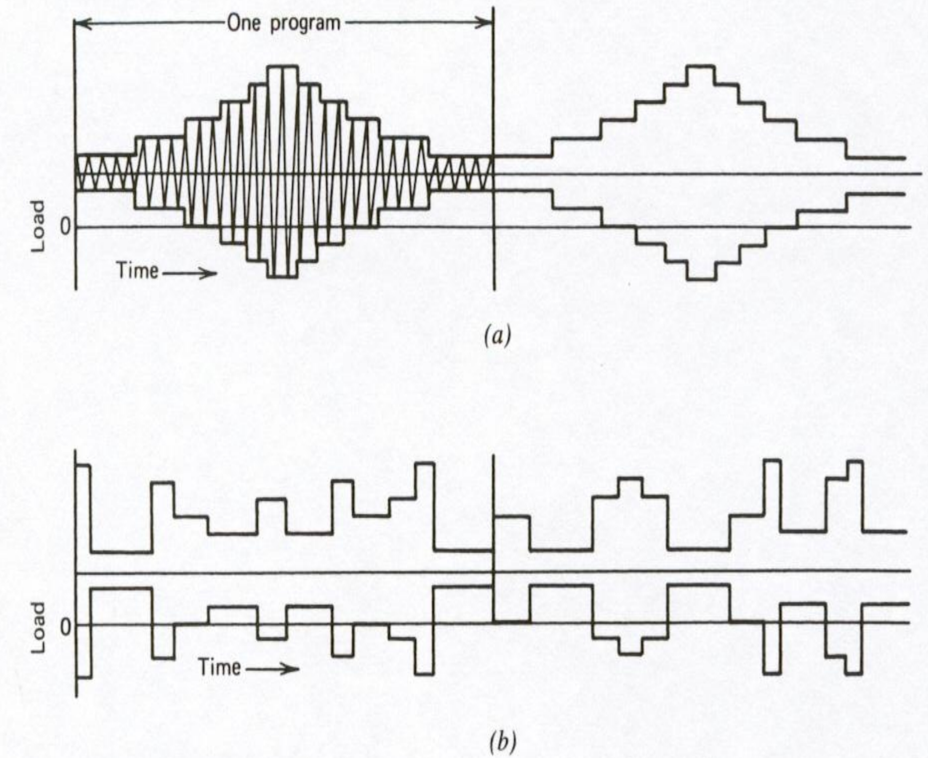


Figure 9.2 Block program load spectra. (*a*) Programmed six load level test. (*b*) Random block program loading.

SPECTRUM LOADS & **CUMULATIVE DAMAGE**

- The term "**cumulative damage**" refers to the fatigue effects of loading events other than constant amplitude cycles.
- The term "**spectrum**" as used in fatigue literature often means a series of fatigue loading events other than uniformly repeated cycles.
- Sometimes spectrum means a listing, ordered by size, of components of irregular sequences, as, for instance, in Table 9.1 for the suspension service history in Fig. 9.1*a*.
- Other parameters, such as maximum and minimum loads, are also used to define the classifications or "boxes" in which the counts of cycles are listed. Boxes in Table 9.1 contain cycle counts for different combinations of stress ranges & means.

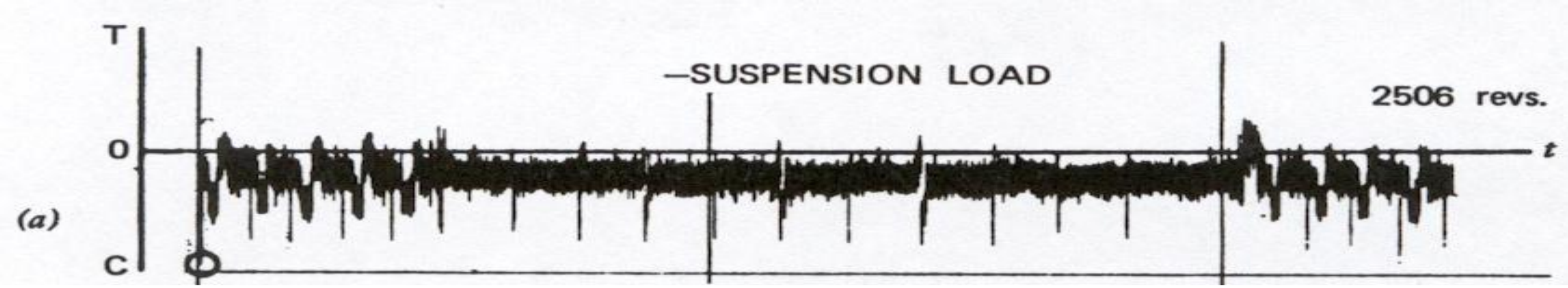


TABLE 9.1 Example of the Number of Cycles at Various Stress Range and Mean Stress Combinations from the Suspension Load History in Fig. 9.1*a* [1]

Stress Range (MPa)	Mean Stress (MPa)													
	−500	−450	−400	−350	−300	−250	−200	−150	−100	−50	0	50	100	150
200	8	16	24	8	6	66	358	104	6	10	6	6	4	2
250	2	16	46	10	4	72	564	118	20	10	10	2	2	6
300	2	8	22	6		36	312	90	14	8	4			
350			4	6		14	148	36	20	6	2			
400			2	10	6	4	64	12	10	6				
450				4		2	20	10	4			2		
500				2	2		8	12				4		
550					4	2	8	2		2				
600				2			2							
650				2	4	4	2		2					
700				4	2									
750			2											
800				2										
850				4	2									
900				4	4									
950				2	4	2								
1000			2		2									
1050					2									
1100														
1150			2											
1200														
1250														
1300														
1350				2										

DAMAGE **QUANTIFICATION**

- One approach to variable load histories uses the concept of damage, defined as the **fraction of life** (also referred to as **cycle ratio**) used up by an event. These fractions are added together; when their sum reaches 1.0 or 100 percent we expect failure. This is the *most common* measure of damage, and is the quantifying measure we use here.

- In addition to the life fraction (or cycle ratio), crack length or crack population, and many other measures have been used to quantify fatigue damage. These include:
  - Metallurgical parameters (size or number of dislocations and spacing or intensity of slipbands)
  - Mechanical parameters (i.e. hardness, stress, strain, stiffness, strain energy)
  - Physical measures (X-radiography, acoustic emission, ultrasonic techniques, magnetic field methods, potential drop, and eddy current techniques)

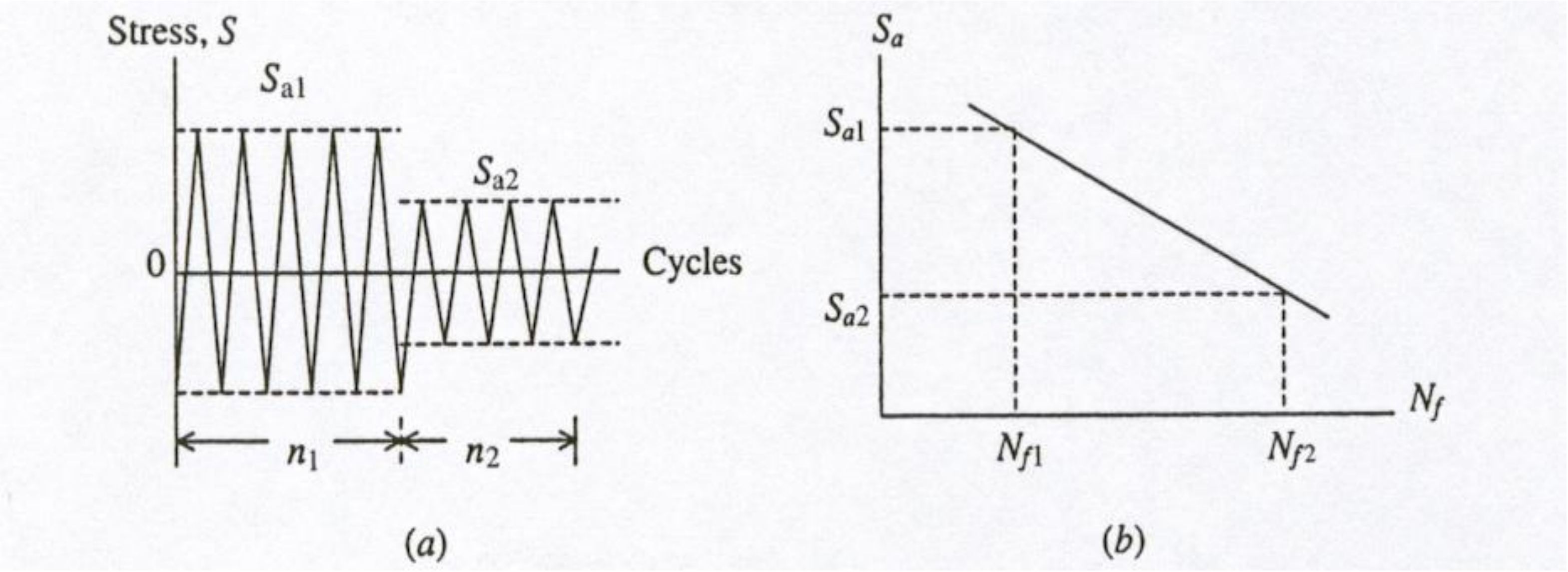
*L. Yang and A. Fatemi, "**Cumulative Fatigue Damage Mechanisms and Quantifying Parameters: A Literature Review**," J. of Testing and Evaluation, Vol. 26, 1998.*

CUMULATIVE DAMAGE THEORIES

- **Palmgren-Miner Linear Damage Rule**:
  - The damage caused by one cycle is defined as $D = 1/N_f$ where N_f is the number of repetitions of this same cycle that equals life to failure.
  - The damage produced by n such cycles is then $nD = n/N_f$.
  - Illustrated in Figure 9.3.

CUMULATIVE DAMAGE THEORIES

- The damaging effect of n_1 cycles at S_{a1} stress amplitude is assumed to be $n_1D_1 = n_1/N_{f1}$, while the damaging effect of n_2 cycles at S_{a2} stress amplitude is assumed to be $n_2D_2 = n_2/N_{f2}$.
- Similarly, the cycle ratio or damage caused by n_i cycles at S_{ai} stress amplitude is $n_iD_i = n_i/N_{fi}$.
- Failure is predicted when the sum of all ratios becomes 1 or 100%.



CUMULATIVE DAMAGE THEORIES

- The following relation expresses the **linear damage rule**, proposed by Palmgren and later again by Miner:

$$\sum \frac{n_i}{N_{fi}} = \frac{n_1}{N_{fi}} + \frac{n_2}{N_{f2}} + \ldots\ldots = 1$$

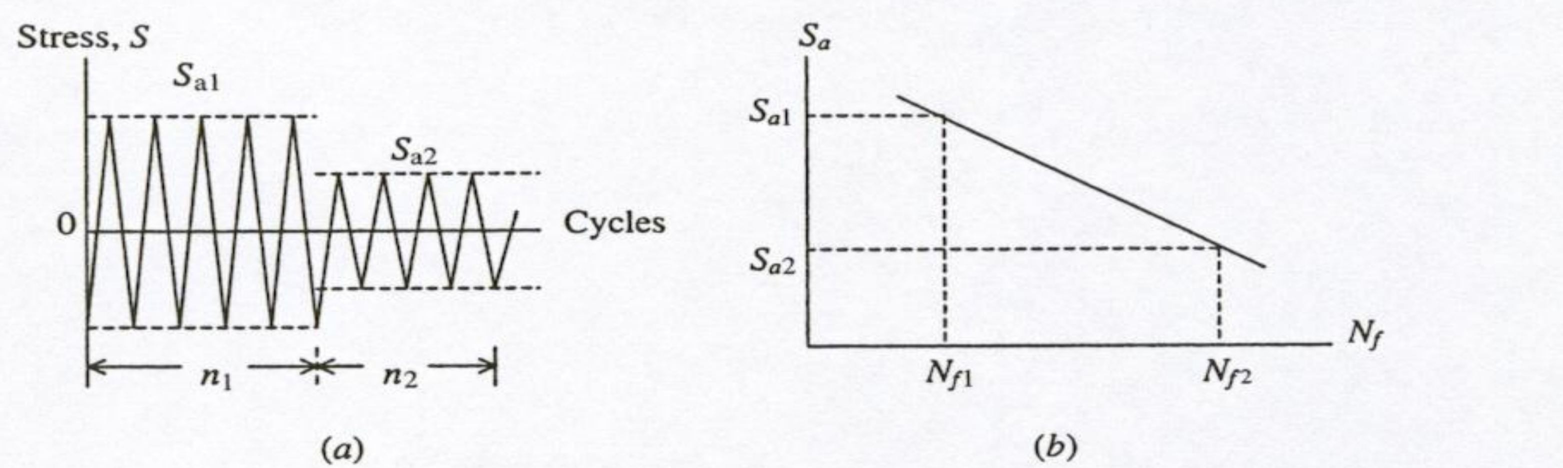


Figure 9.3 Constant amplitude stress blocks and S–N curve. (*a*) Constant amplitude stress blocks. (*b*) S–N curve.

CUMULATIVE DAMAGE THEORIES

- **Example** of application to ball bearings:
  - Tests show the median life of a certain model bearings operating at high frequency to be:
    - $2x10^8$ cycles under 1 kN load and
    - $3x10^7$ cycles under 2 kN load.

How many cycles can we expect the bearing to last if the load is 1 kN 90% of the time and 2 kN 10% of the time?

If the total applied cycles is *n*, the number of cycles at the 1 kN and 2 kN loads are $n_1 = 0.9n$ and $n_2 = 0.1n$, respectively. The total damage done will be:

$$D = \frac{n_1}{N_{fi}} + \frac{n_2}{N_{f2}} = \frac{0.9n}{2x10^8} + \frac{0.1n}{3x10^7} = 7.83x10^{-9}\,n$$

Failure is expected when D = 1. The expected life is then

$1/D = 1/(7.83x10^{-9}) = 1.3x10^8$ cycles.

CUMULATIVE DAMAGE THEORIES

- Equation 9.1 for linear damage rule is also used with other fatigue curves such as load-life or ε-N curves.

- The assumption of linear damage is open to many objections. For example,
  - sequence and interaction of events may have major influences on life
  - the rate of damage accumulation may depend on the load amplitude
  - experimental evidence often indicates that $\sum n_i/N_{fi} \neq 1$ for a low-to-high or a high-to-low loading sequence.

- Even though the linear damage rule ignores these effects, it is most widely used because of its simplicity and the fact that none of the other proposed methods achieves better agreement with data from many different tests.

CUMULATIVE DAMAGE THEORIES

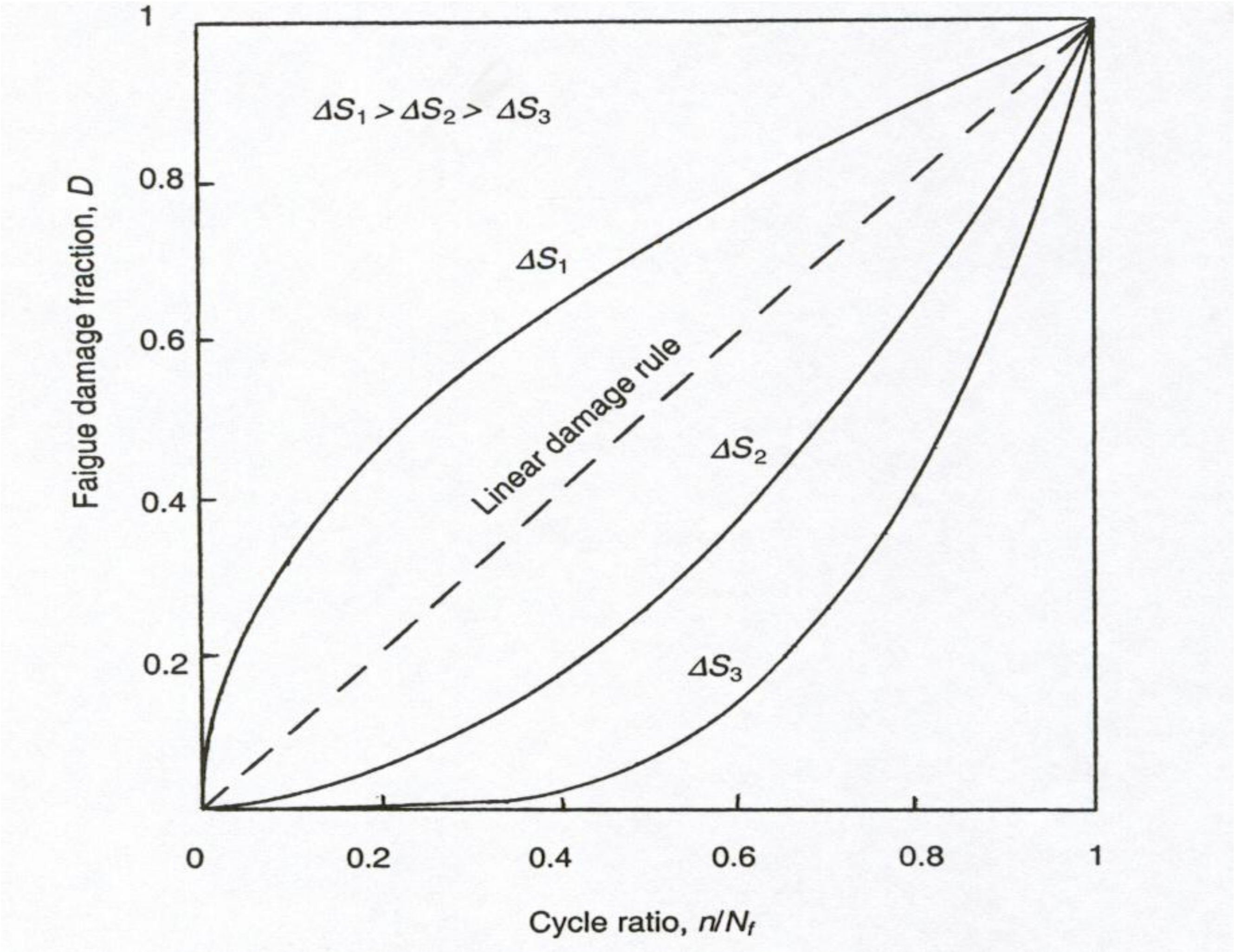
- **Nonlinear Damage Theories**
  - To remedy the deficiencies with the linear damage assumption, many nonlinear cumulative damage rules have been proposed.

    A. Fatemi and L. Yang, "Cumulative Fatigue Damage and Life Prediction Theories: A Survey of the State of the Art for Homogeneous Materials," Int. J. Fatigue, Vol. 20, No. 1, 1998.

  - These theories account for the nonlinear nature of fatigue damage accumulation by using nonlinear relations such as $D = \sum(n_i/N_{fi})^{\alpha_i}$ where the power α_i depends on the load level (see Fig. 9.4).

  - Though many nonlinear damage models have been developed, unfortunately none can encompass many of the complicating factors encountered during complex variable amplitude loading.

Linear damage rule and $D = \sum(n_i/N_{fi})^{\alpha_i}$ nonlinear rule at three stress levels in a plot of fatigue damage versus cycle ratio.



LOAD INTERACTION & SEQUENCE EFFECTS

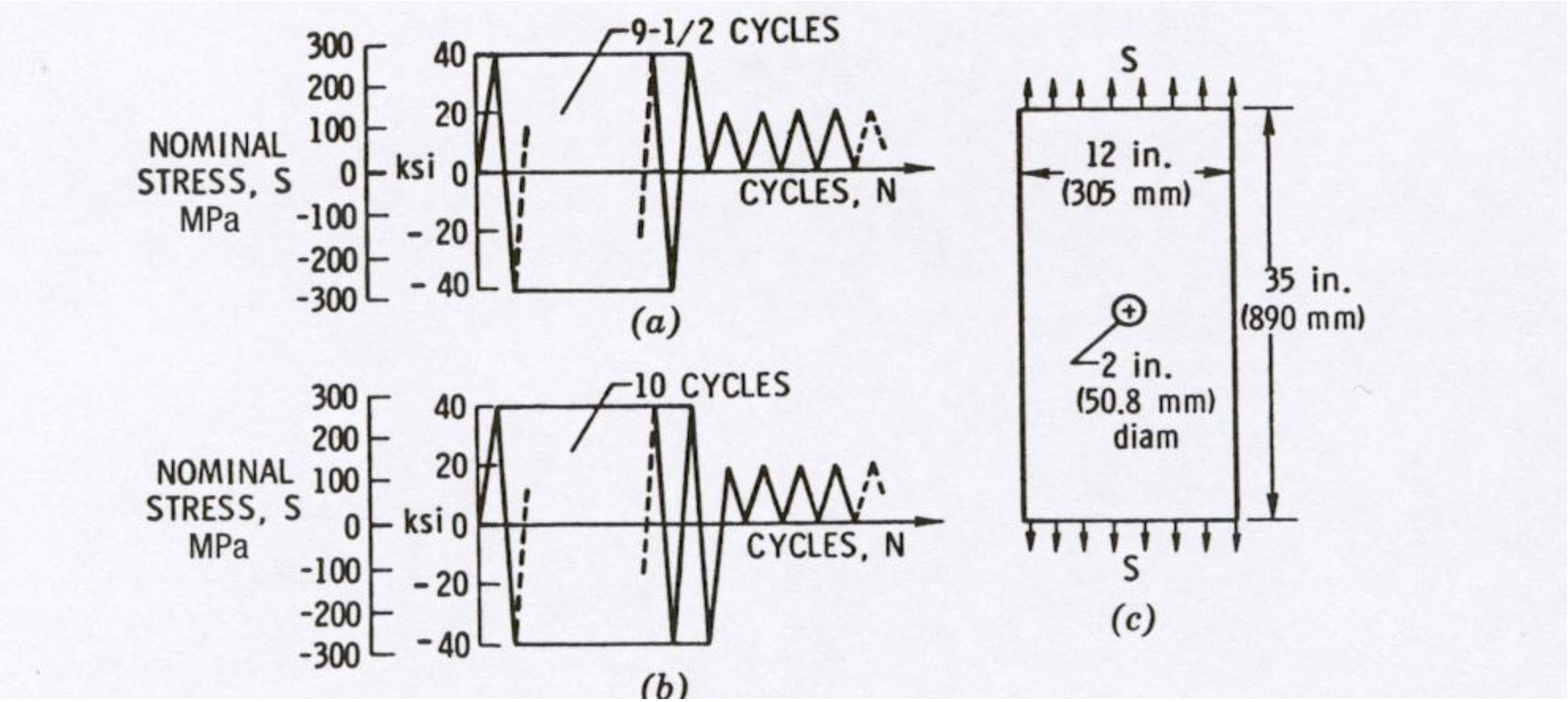
- **Sequence effects** exist both in the early stages (crack nucleation and microcrack growth) and in the later stages (macrocrack growth) of fatigue.

  - Figure 9.5: Sequence effects on crack nucleation

  - Figs. 9.6 & 9.7: Sequence effects on crack growth

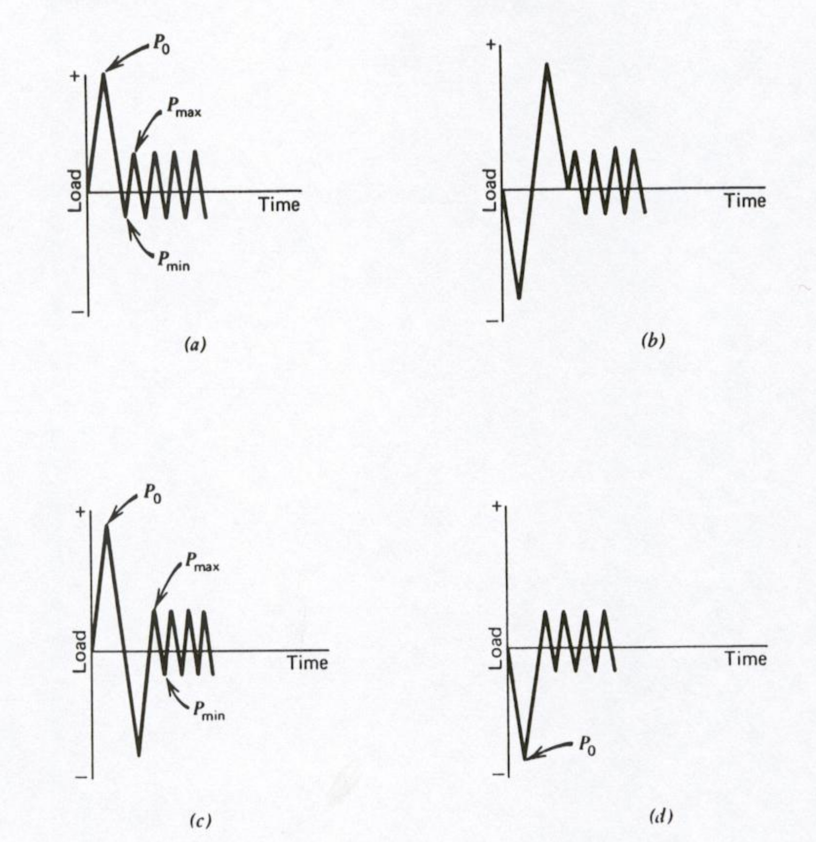
Fig. 9.5: Sequence effects on **crack nucleation**

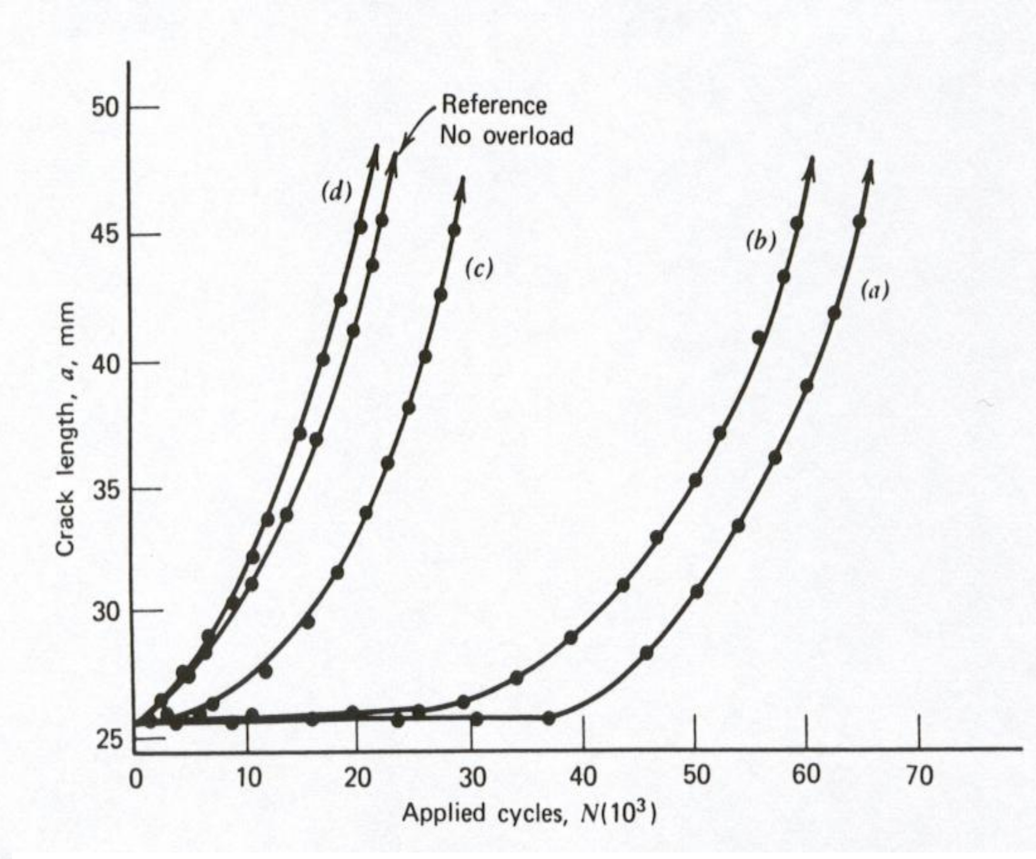
- The life of a specimen with a hole was ***460,000*** cycles of low load after 9.5 cycles of high load, but only ***63000*** cycles after 10 cycles of high load.
- This difference is explained by the residual stresses remaining from the high loads.



Figs. 9.6 & 9.7: Sequence effects on **crack growth**

- Fatigue crack growth from different load patterns.
- Yielding and the resulting residual stresses and crack closure near the crack tip are the main causes for these effects.

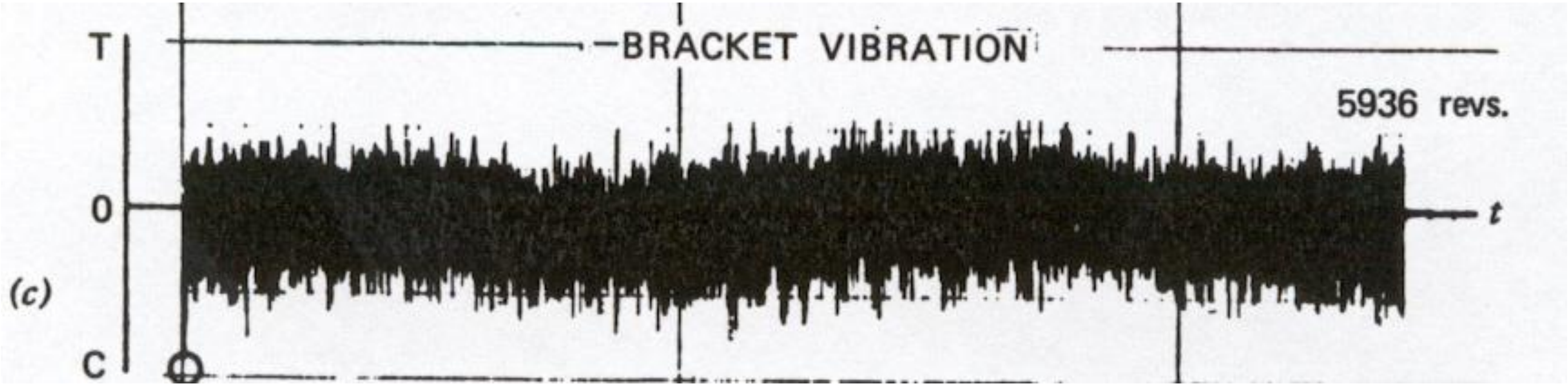




LOAD INTERACTION & SEQUENCE EFFECTS

- Many service histories are such that sequence effects either cancel each other or are entirely unpredictable.

- A few **qualitative rules** based on experience can be stated as follows:
  - If the loading is random with widely varying amplitudes at similar frequencies there will be no definable sequence (I.e. Figure 9.1*c*).
  - If the loading history shows infrequent high loads in one direction, as for instance in the ground-air-ground cycle of aircraft, one should expect sequence effects.
  - Infrequent tensile overloads produce retardation of crack growth or crack arrest. Compressive overloads large enough to produce yielding can produce the opposite effect.



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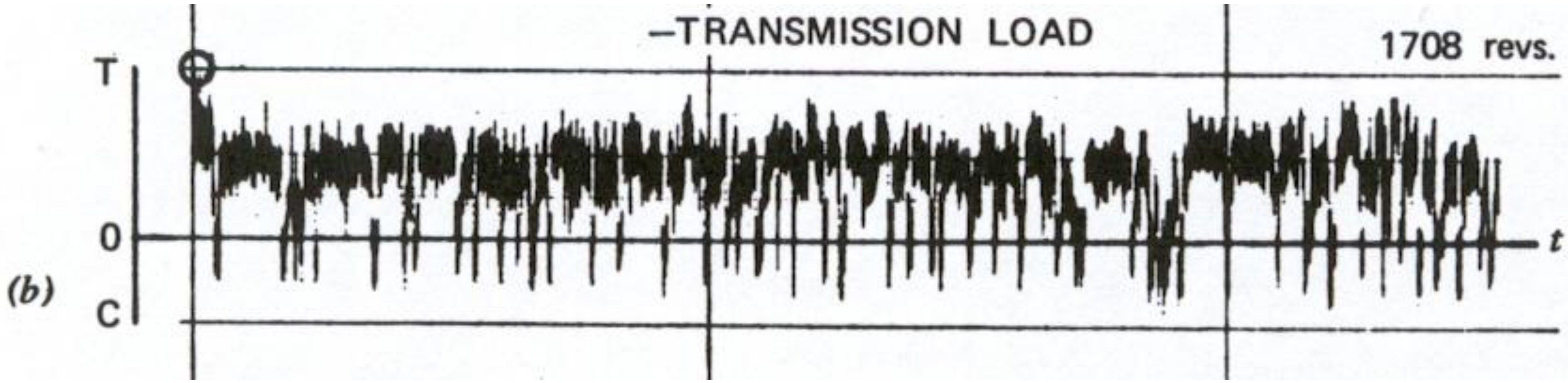
LOAD INTERACTION & SEQUENCE EFFECTS

- When the future load history can be predicted fairly well or is prescribed by the customer, one can follow it reversal by reversal, either by test programs applied to the part or analytically.

- Sequence effects must be expected when between many minor ranges there are a few major deviations that always end in the same way in coming back to the minor ranges.

LOAD INTERACTION & SEQUENCE EFFECTS

- Figure 9.l*b* shows a history that could be expected to show sequence effects because between the 1600 minor ranges there are about 100 major deviations that always come back from high compression.
  - Calculations with and without considering sequence effects were made for two materials and three load levels and these were compared with fatigue test results for a **notched keyhole specimen**).
  - There were no significant differences for the softer material, Man-Ten, nor for the lowest load level, which produced fatigue lives of about 40 000 blocks of 1708 reversals.



LOAD INTERACTION & SEQUENCE EFFECTS

- For the harder material, RQC-100, at higher load levels the results were as listed below:

Test lives	22-30	269-460	Blocks
Calculated without sequence	69	1 300	Blocks
Calculated with sequence	10	170	Blocks

The numbers quoted above are for lives to the appearance of obvious cracks taken as 2.5 mm long.

CYCLE COUNTING METHODS

CYCLE COUNTING METHODS

- The object of all cycle counting methods is to compare the effect of variable amplitude load histories to fatigue data and curves obtained with simple constant amplitude load cycles.

- Applying linear damage rule $\sum n_i/N_{fi} = 1$ requires knowledge of mean and amplitude of stress or strain to which the damaging event is compared.

- Different counting methods can change resulting predictions significantly.

- All good counting methods must count a cycle with the range from the highest peak to the lowest valley and seek to count other cycles in a manner that maximizes the ranges that are counted (i.e. intermediate fluctuations are less important than the overall differences between high points and low points).

- All good counting methods count every part of every overall range once.

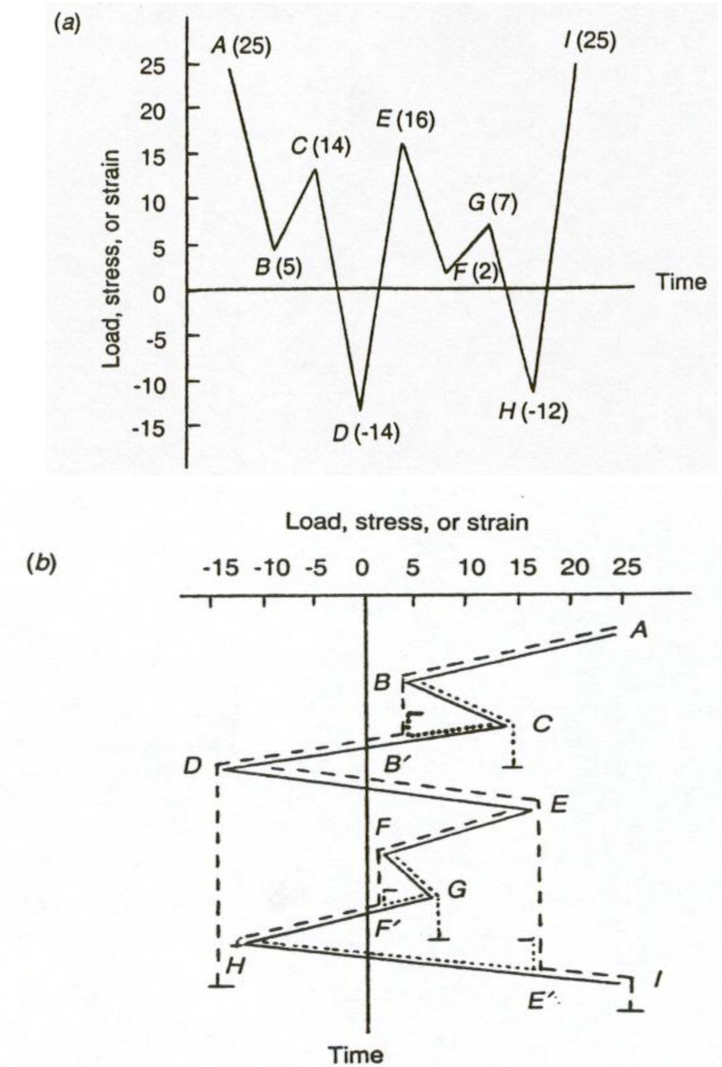
CYCLE COUNTING METHODS

- Several counting methods that achieve this objective are:
  - **Rainflow Method** (the most popular and probably the best method of cycle counting)
  - Variations of rainflow counting and similar methods:
    - range-pair counting method
    - racetrack counting method
  - Other cycle counting methods such as:
    - level-crossing counting method
    - peak counting method

RAINFLOW COUNTING METHOD

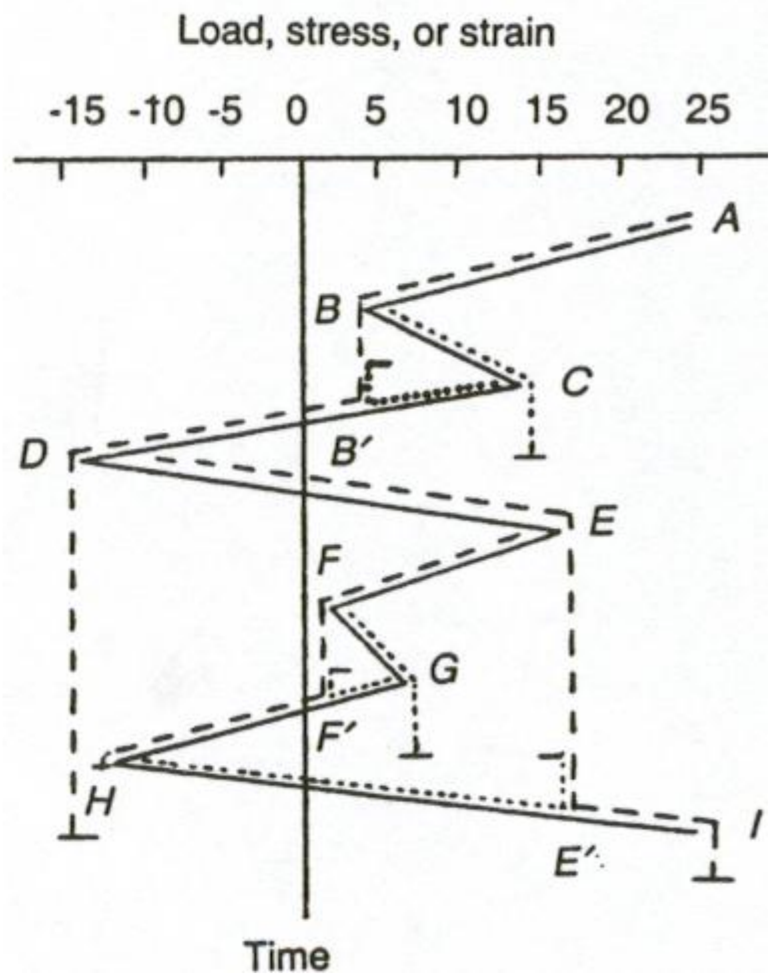
- With the load-time, stress-time, or strain-time history plotted such that the time axis is vertically downward, we can think of the lines going horizontally from a reversal to a succeeding range as **rain flow**ing down a roof represented by the history of peaks and valleys.

- The operation of the rainflow method is shown in Fig. 9.9 for a history consisting of four peaks and four valleys.



RAINFLOW COUNTING METHOD

- The rules are:
  - Rearrange the history to start with either the highest peak or the lowest valley.
  - If starting from the highest peak, go down to the next reversal. The rain flow runs down and continues, unless either the magnitude of the following peak is equal to or larger than the peak from which it initiated, or a previous rain flow is encountered.
  - Repeat the same procedure for the next reversal & continue these steps to the end.
  - Repeat the procedure for all the ranges and parts of a range that were not used in previous steps.

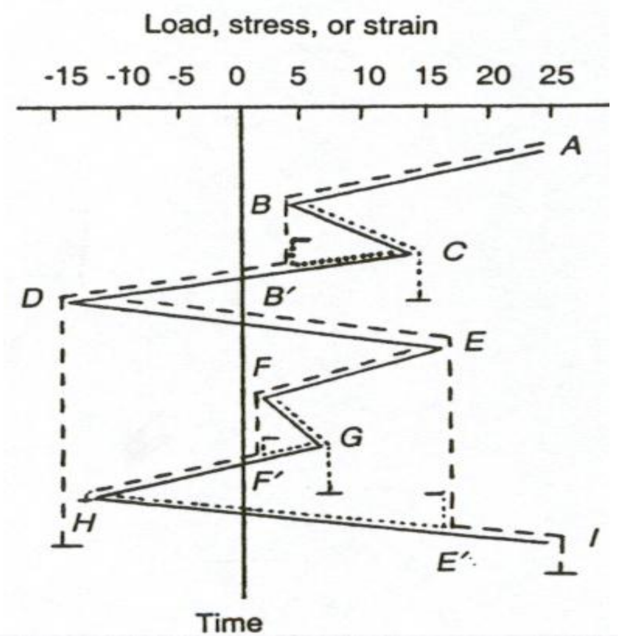


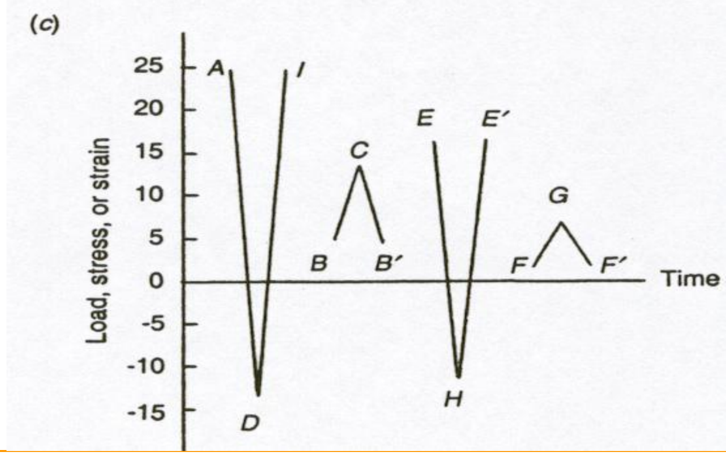
RAINFLOW COUNTING METHOD

- It should be noted that
  - every part of the load history is counted only once,
  - the counted half cycles always occur in pairs of equal magnitude, resulting in full cycles.

  - In the example history, the resulting range and mean values are as follows:

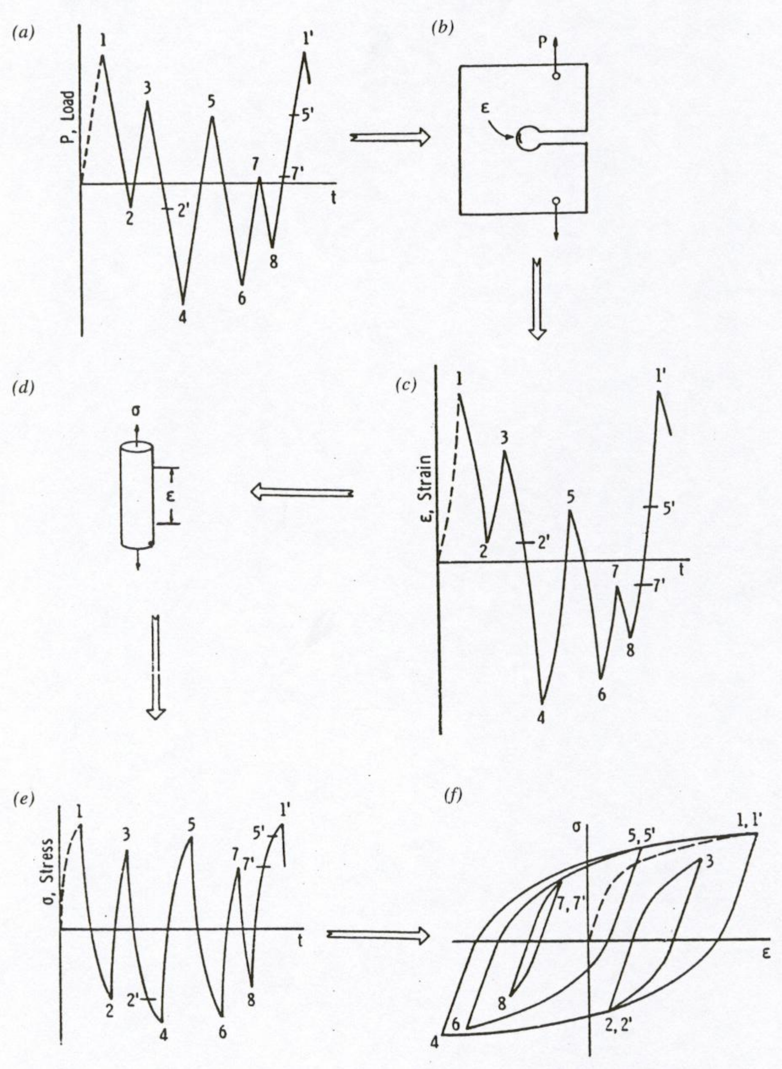
Cycle	Max	Min	Range	Mean
A-D-I	25	-14	39	5.5
B-C-B′	14	5	9.5	4.5
E-H-E′	16	-12	28	2
F-G-F′	7	2	5	4.5





RAINFLOW COUNTING METHOD

- An advantage of rainflow counting is when it is used with notch strain analysis.
  - Note that the rainflow counting results in **closed hysteresis loops**, with each closed loop representing a counted cycle. Therefore, the closed hysteresis loops can also be used to obtain the cycle counting.
  - The tips of the largest hysteresis loop are at the largest tensile and compressive loads in the load history (points 1 and 4).
  - The **notch strain-time history** (Fig. *c*) is quite different from the corresponding notch stress-time history (Fig. *e*).
  - During each segment of the loading the material "remembers" its prior deformation (called **material memory**).



RAINFLOW COUNTING METHOD

- The damage from each counted cycle can be computed from the strain amplitude and mean stress for that cycle as soon as it has been identified in the counting procedure. The corresponding reversal points can then be discarded.

- A computer program that accomplishes rainflow cycle counting applied to a complex history such as that in Fig. 9.1*a* results in a table of ranges and means shown in Table 9.1.

- Data acquisition systems for real time rainflow counting of strain gage signals are commercially available.

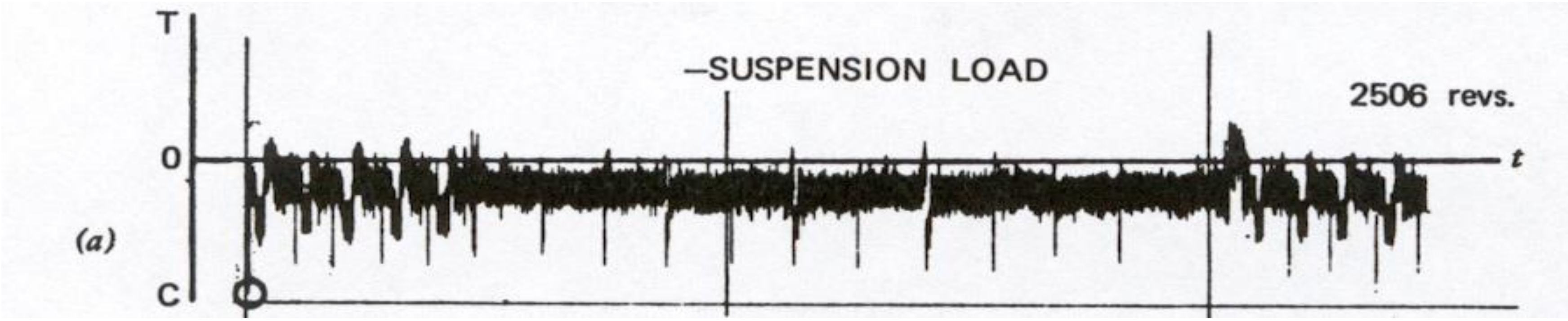


TABLE 9.1 Example of the Number of Cycles at Various Stress Range and Mean Stress Combinations from the Suspension Load History in Fig. 9.1*a* [1]

Stress Range (MPa)	Mean Stress (MPa)													
	−500	−450	−400	−350	−300	−250	−200	−150	−100	−50	0	50	100	150
200	8	16	24	8	6	66	358	104	6	10	6	6	4	2
250	2	16	46	10	4	72	564	118	20	10	10	2	2	6
300	2	8	22	6		36	312	90	14	8	4			
350			4	6		14	148	36	20	6	2			
400			2	10	6	4	64	12	10	6				
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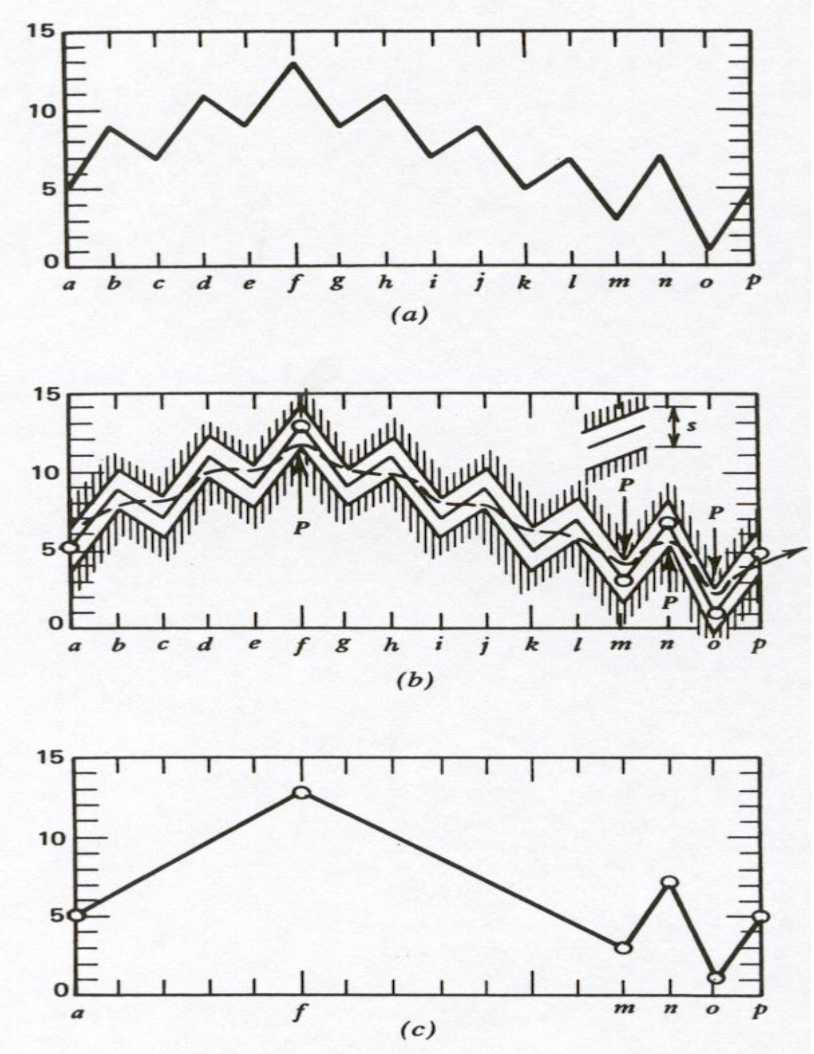
1350				2										

RACETRACK COUNTING METHOD

- The method of eliminating smaller ranges is indicated in Fig. *b*.
- A "racetrack" or width *S* is defined, bounded by "fences" that have the same profile as the original history.
- Only those reversal points are counted at which a "racer" would have to change from upward to downward, as at *f* and *n*, or vice versa as at *m* and *o*.
- The width *S* of the track determines the number of reversals that will be counted.
- The original history in Fig. *a* is condensed to the history in Fig. *c*.



RACETRACK COUNTING METHOD

- The object of this method is to condense a long complex history of reversals.
- The condensed history includes the sequence of events, which may be important if yielding produces residual stresses that remain active for many succeeding reversals.
- This method is useful for **condensing histories** to those few events, perhaps the 10 percent of events that do most of the damage, which usually account for more than 90 percent of all calculated damage.
- The condensed histories accelerate testing and computation and permit focusing of attention on a few significant events.

LIFE ESTIMATION USING
S-N APPROACH

LIFE ESTIMATION USING **S-N APPROACH**

- This method neglects sequence effects.
- An *S-N* curve is used as primary input.
- Materials and service conditions for which a fatigue limit may exist under constant amplitude cycling, may no longer exhibit a fatigue limit under variable amplitude loading, if the largest loads are above the fatigue limit.
- Load ranges and mean loads of the given load history are counted by rainflow or another cycle counting method.

LIFE ESTIMATION USING S-N APPROACH

- In many cases 10 percent of all greatest overall ranges will do more than 90 percent of the damage. These greatest overall ranges can easily be picked out from a plot of peaks and valleys.
- Next the load ranges and means must be converted to nominal stress ranges and means.
- Finally, the damage expected from each of the stress ranges is calculated from the *S-N* curve as $1/N_{fi}$, and the damages are added. The ratio of 1/(sum of damages) is the number of times we expect the given history to be endured until failure occurs.

LIFE ESTIMATION USING **S-N APPROACH**

- The procedure is summarized in a **flow chart**

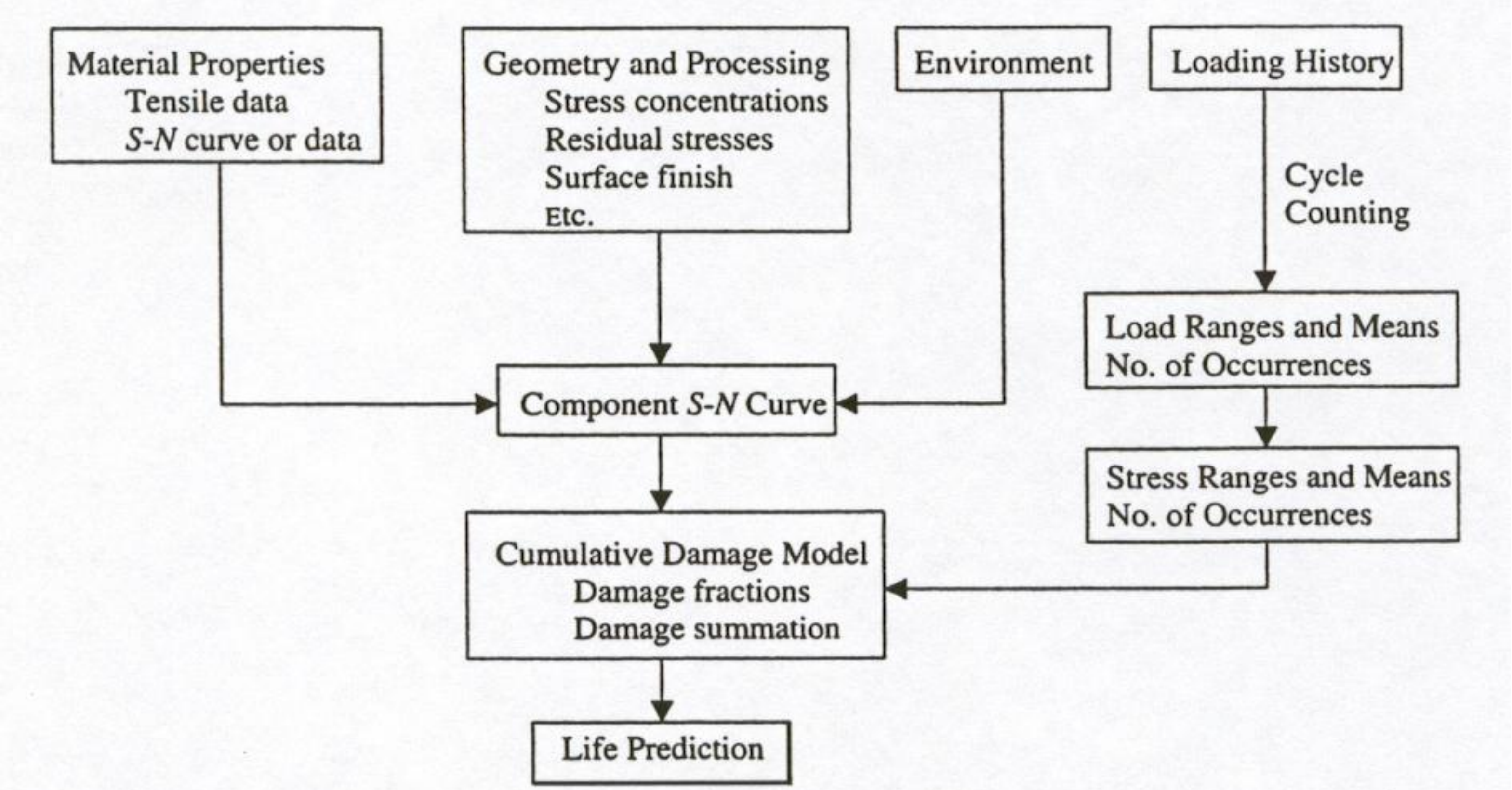


Figure 9.15 Sequential steps in predicting fatigue life based on the S–N approach.

LIFE ESTIMATION USING S-N APPROACH: **EXAMPLE**

- A round shaft made of RQC-100 steel is repeatedly subjected to the shown block of nominal axial stress history.

  (a) If the shaft is **smooth** with a polished surface finish, how many blocks of this stress history can be applied before failure is expected?

  (b) Repeat part (a) if the shaft has a circumferencial **notch** with a notch root radius of 1 mm and $K_t = 2$.

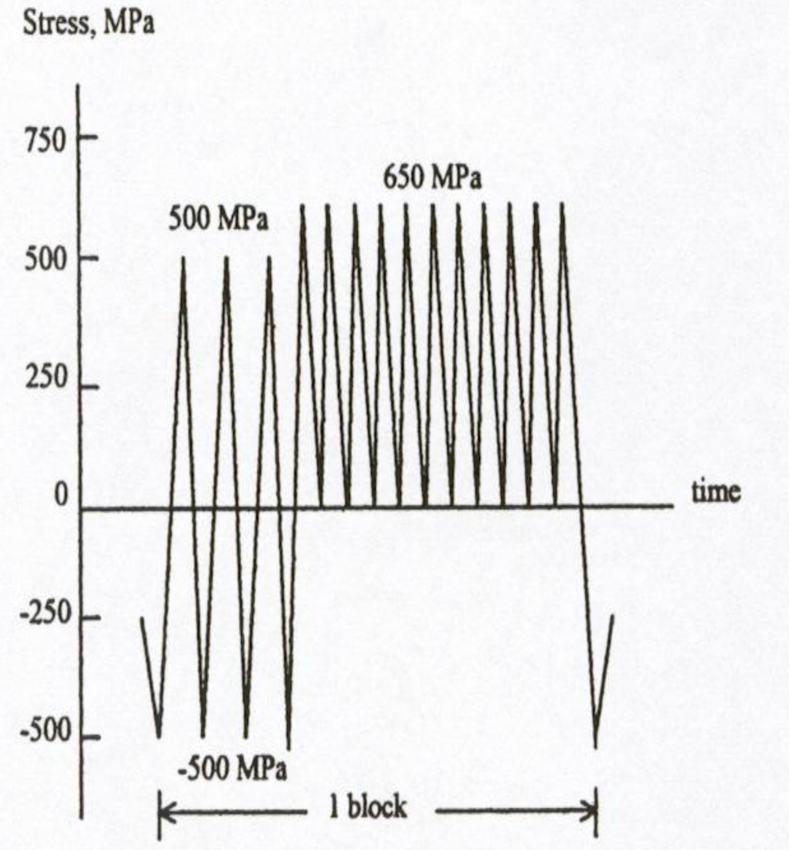


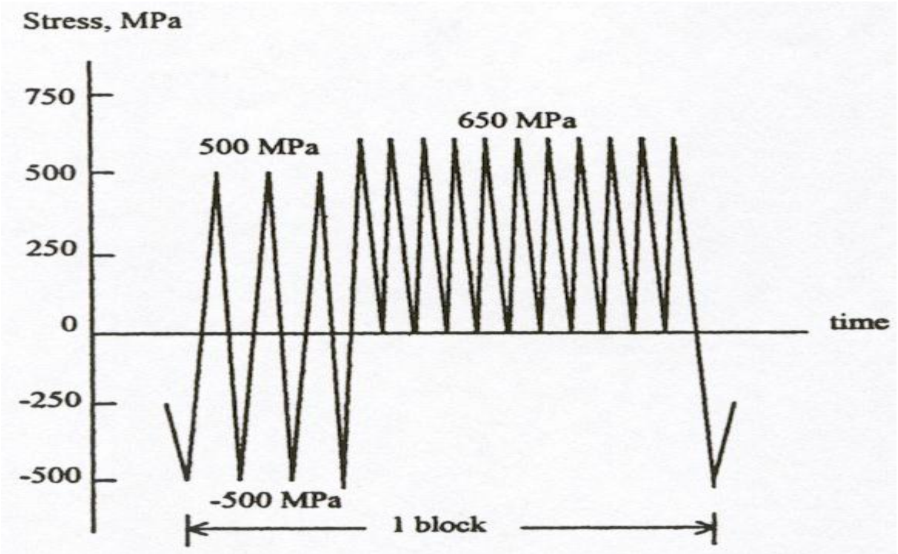
Figure 9.16 Nominal axial stress history applied to a shaft

LIFE ESTIMATION USING S-N APPROACH: **EXAMPLE**

- The loading history consists of three constant amplitude load segments summarized as follows:

Segment	S_{min} (MPa)	S_{max} (MPa)	S_a (MPa)	S_m (MPa)	n
1	-500	500	500	0	3
2	-500	650	575	75	1
3	0	650	325	325	10

Note that load segment 2 may not be apparent, but in fact it is the most damaging cycle in the load history. A rainflow count will identify this cycle.



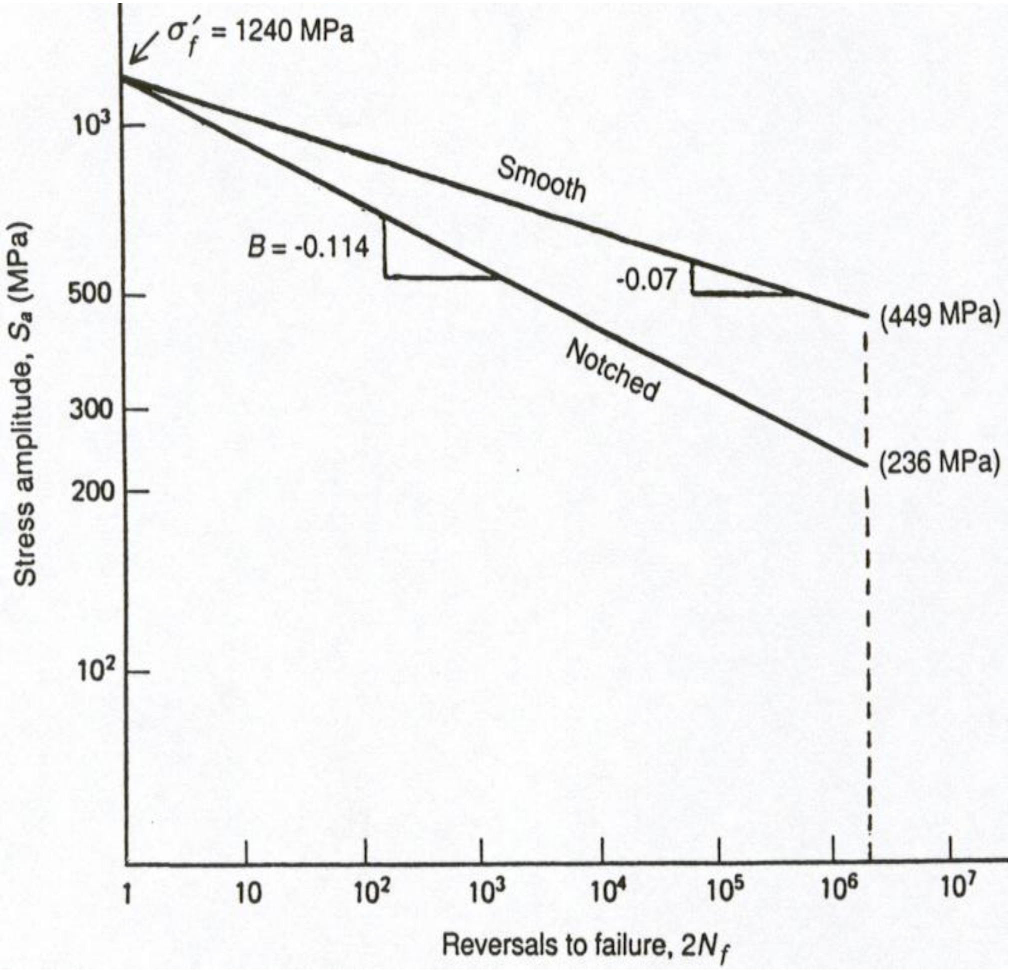
LIFE ESTIMATION USING S-N APPROACH: **EXAMPLE**

- **(a)** Using the material fatigue properties from Table A.2, the *S-N* curve for RQC-100 steel is given by:

  $$S_{Nf} = \sigma_f' (2N_f)^b = 1240\ (2N_f)^{-0.07}$$

  where S_{Nf} is the fully reversed (R = -1) fatigue strength at $2N_f$ reversals.

- The *S-N* curve for fully reversed loading is shown.



LIFE ESTIMATION USING S-N APPROACH: **EXAMPLE**

- Since the loadings in the second and third segments are not completely reversed, we can account for the **mean stress** by using one of the mean stress correction parameters, such as modified Goodman:

$$\frac{S_a}{S_{Nf}} + \frac{S_m}{S_u} = 1$$

S_u = 931 MPa from Table A.3. The equivalent fully reversed stress amplitude, S_{Nf}, at the 75 MPa and 325 MPa mean stress levels are now computed as:

At S_m = 75 MPa: $\frac{S_a}{S_{Nf}} + \frac{75}{931} = 1$ or $S_{Nf} = 1.088\ S_a$

At S_m = 325 MPa: $\frac{S_a}{S_{Nf}} + \frac{325}{931} = 1$ or $S_{Nf} = 1.536\ S_a$

LIFE ESTIMATION USING S-N APPROACH: **EXAMPLE**

- We can now obtain cycles to failure, N_{fi} at each equivalent fully reversed stress amplitude, S_{Nfi} and compute damage fraction for each load segment, n/N_f, summarized as follows:

Segment	S_a (MPa)	S_{Nf} (MPa)	N_f	n	n/N_f
1	500	500	215 770	3	1.4×10^{-5}
2	575	625	8 815	1	1.13×10^{-4}
3	325	499	219 630	10	4.6×10^{-5}
				$\sum n_i/N_{fi}$ =	1.73×10^{-4}

- The expected life is calculated as the reciprocal of $\sum n_i/N_{fi}$ or $1/(1.73 \times 10^{-4}) = 5\ 780$ blocks.

LIFE ESTIMATION USING S-N APPROACH: **EXAMPLE**

- **(b)** Repeat part (*a*) if the shaft has a circumferencial **notch** with a notch root radius of 1 mm and a stress concentration factor of $K_t = 2$.

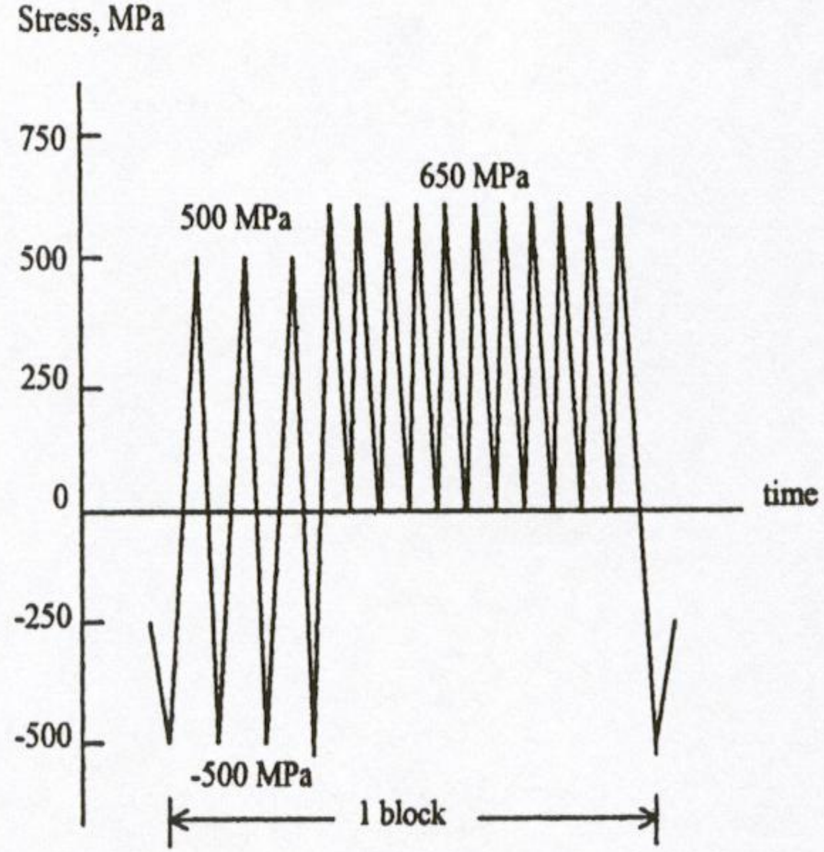


Figure 9.16 Nominal axial stress history applied to a shaft.

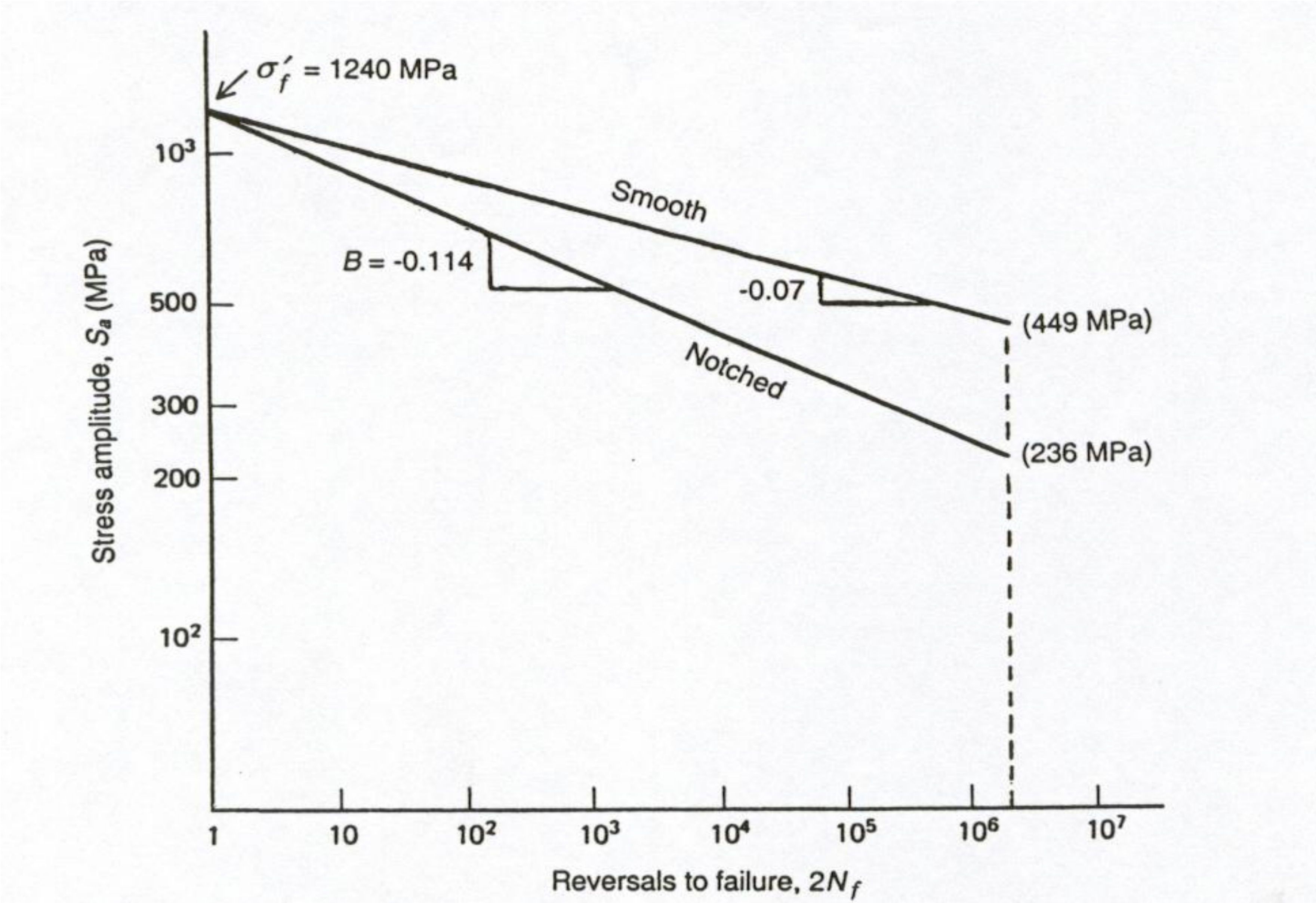
LIFE ESTIMATION USING S-N APPROACH: **EXAMPLE**

(***b***) For the **notched** shaft, we first obtain the fatigue notch factor, K_f. Using Peterson's equation:

$$a = 0.0254\left(\frac{2070}{S_u}\right)^{1.8} = 0.0254\left(\frac{2070}{931}\right)^{1.8} = 0.11$$

$$K_f = 1 + \frac{K_t - 1}{1 + a/r} = 1 + \frac{2-1}{1 + 0.11/1} = 1.90$$

- The *S-N* line equation for fully reversed loading (R = -1) of the notched shaft is obtained by connecting σ_f' = 1240 MPa at 1 reversal to S_f/K_f at $2\text{x}10^6$ reversals with:
  $S_f = 1240\ (2\text{x}10^6)^{-0.07} = 449$ MPa, and $S_f/K_f = 449/1.90 = 236$ MPa
- The S-N line for fully reversed loading (R = -1) of the notched shaft is also shown in Fig. 9.17.


$\sigma_f' = 1240$ MPa
10^3
500
300
200
10^2
Stress amplitude, S_a (MPa)
Smooth
$B = -0.114$
-0.07
(449 MPa)
Notched
(236 MPa)
1
10
10^2
10^3
10^4
10^5
10^6
10^7
Reversals to failure, $2N_f$

LIFE ESTIMATION USING S-N APPROACH: **EXAMPLE**

- To obtain the equivalent completely reversed net alternating stress, S_{Nf}, for the second and third load segments where mean stress exists, we can again use the modified Goodman equation, as in part (*a*). We then calculate damage fraction for each load segment as follows:

Segment	S_a (MPa)	S_{Nf} (MPa)	N_f	n	n/N_f
1	500	500	1 442	3	2.1×10^{-3}
2	575	625	202	1	4.9×10^{-3}
3	325	499	1 458	10	6.9×10^{-3}
				$\sum n_i/N_{fi} =$	0.0139

The expected life is calculated as 1/(0.0139) = 72 blocks. It should be remembered that the life calculated is to the nucleation or formation of a crack between 0.25 mm and 5 mm in length.

LIFE ESTIMATION USING

ε-N APPROACH

LIFE ESTIMATION USING ε-N APPROACH

- When the load history contains large overloads, **significant plastic deformation** can exist, particularly at stress concentrations, and load sequence effects can be significant.

- For these cases, the **strain-life approach** is generally a superior approach to the stress-life approach for cumulative damage analysis.

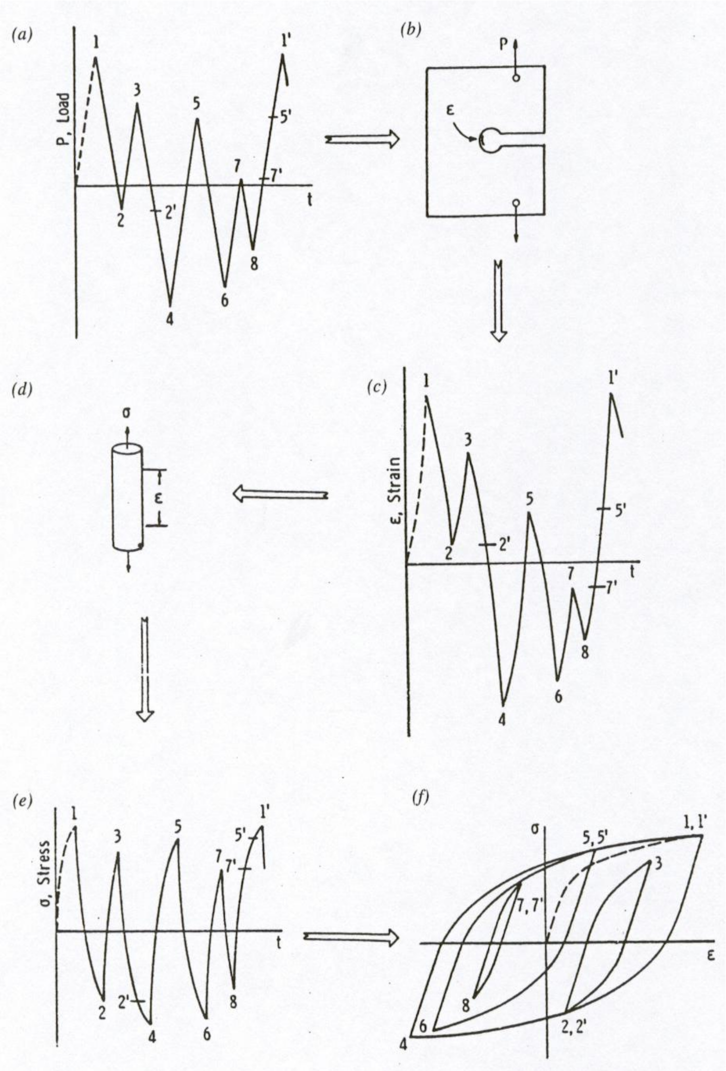
- Application of the strain-life approach requires material strain-life curve.

- For a notched member, notch strain analysis typically with an analytical model such as Neuber, Glinka, or linear rules, along with the cyclic stress-strain curve of the material are used to relate nominal stresses and strains to notch stresses and strains.

- The effect of mean or residual stresses is also accounted for by using one of the mean stress correction parameters.

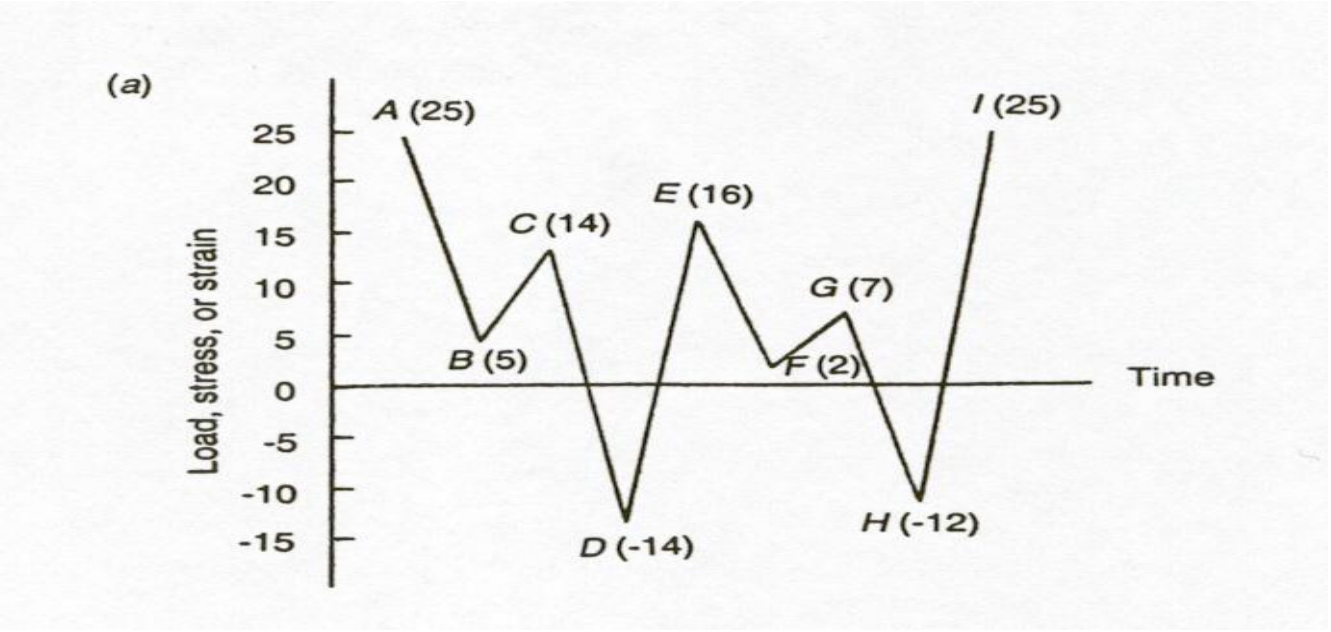
LIFE ESTIMATION USING ε-N APPROACH

- The following procedure is followed:
  - The load history is converted to **nominal stress history**, which is then applied to the component.
  - Using the material cyclic stress-strain relation, and for a notched member a notch strain analysis model (e.g. Neuber's rule), the applied nominal stress history results in the **notch strain history** and stress-strain hysteresis loops
  - The **strain amplitude** and **mean stress** can now be obtained for each rainflow counted cycle from the notch stress and strain history or from the hysteresis loops.
  - Knowing the strain amplitude and mean stress, **damage fraction** corresponding to each cycle, $1/N_{fi}$ is then computed and damages are added to predict failure.



LIFE ESTIMATION USING ε-N APPROACH: **EXAMPLE**

- The notched shaft in Part (*b*) of the example problem in Section 9.6 is subjected to repeated blocks of the variable amplitude net nominal stress in Fig. 9.9, where each unit in Fig. 9.9 is equivalent to 20 MPa. What is the expected life to appearance of a small crack, according to the strain-life approach?



LIFE ESTIMATION USING ε-N APPROACH: **EXAMPLE**

- Cyclic stress-strain properties (K' and n') and strain-life properties (σ_f', b, ε_f', and c) for RQC-100 steel are given in Table A.2.

- The load history is already rainflow counted in Section 9.5.1, as shown in Fig. 9.9*c*.

Cycle	Max	Min	Range	Mean
A-D-I	25	-14	39	5.5
B-C-B'	14	5	9.5	4.5
E-H-E'	16	-12	28	2
F-G-F'	7	2	5	4.5

- To convert the applied net nominal stress history to the notch stress-strain history we choose to use Neuber's rule with $K_f = 1.9$, as calculated for the example problem in Section 9.6.

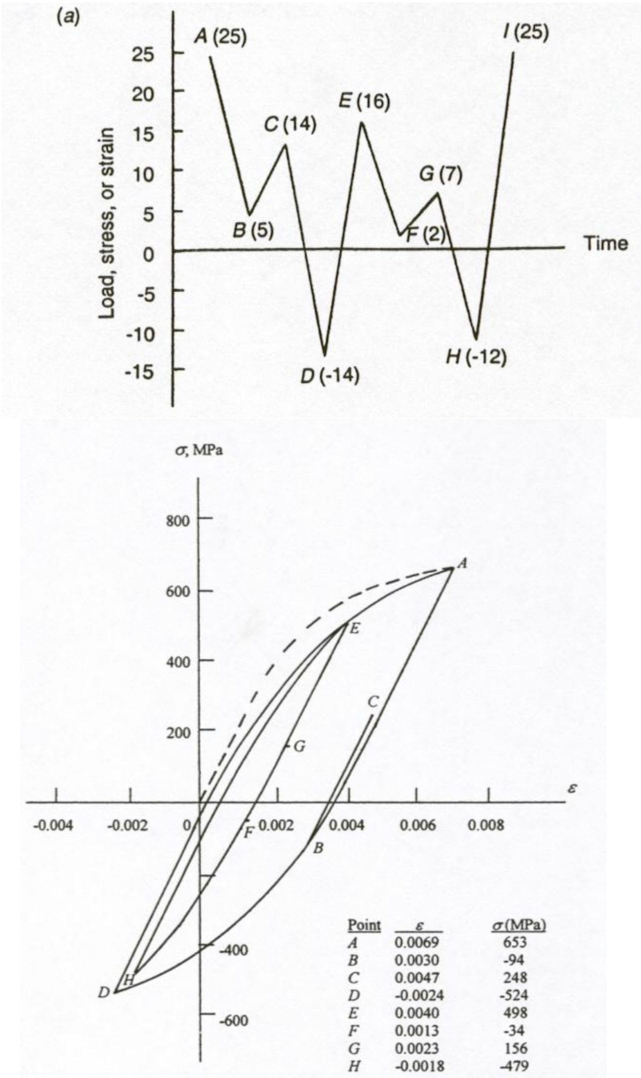
LIFE ESTIMATION USING ε-N APPROACH: **EXAMPLE**

- Initial loading to point *A*, which is the highest load in the history, is at a net nominal stress of *S* = 25x20 = 500 MPa. This stress level is 83% of the cyclic yield strength (S_y' = 600 MPa) and, therefore, the nominal behavior is elastic ($e = S/E$).
- Notch root stress and strain at point *A* are then calculated from Neuber's rule and the cyclic stress-strain equation as follows:

$$\varepsilon\,\sigma = \frac{(K_f\,S)^2}{E} = \frac{(1.9\,x500)^2}{200000} = 4.51$$

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{1/n'} = \frac{\sigma}{200000} + \left(\frac{\sigma}{1434}\right)^{1/0.14}$$

resulting in ε = 0.0069 and σ = 653 MPa (see dashed line). Notch deformation at point *A* is inelastic, as the stress level at this point is above the cyclic yield strength.



LIFE ESTIMATION USING ε-N APPROACH: **EXAMPLE**

- For loading from point A to point B, we use the hysteresis loop equation, with a nominal stress range ($\Delta S_{A\text{ to }B}$ = (25-5)x20 = 400 MPa, as follows:

$$\Delta\varepsilon\,\Delta\sigma = \frac{(K_f\,\Delta S)^2}{E} = \frac{(1.9\,x\,400)^2}{200000} = 2.89$$

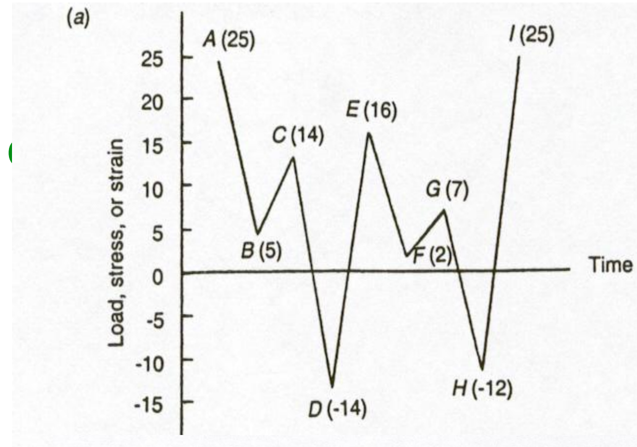
$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} = \frac{\Delta\sigma}{200000} + 2\left(\frac{\Delta\sigma}{2868}\right)^{1/0.14}$$

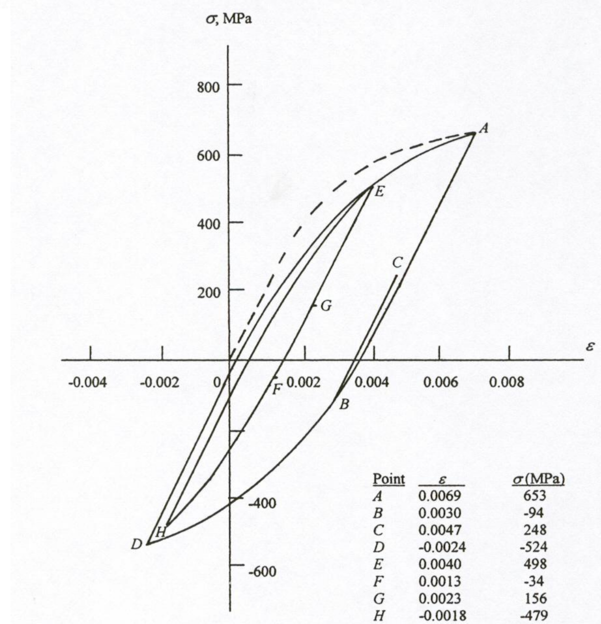
resulting in $\Delta\varepsilon$ = 0.0039 and $\Delta\sigma$ = 747 Mpa.

- Now we can compute **notch root stress and strain** at point B in the loading:

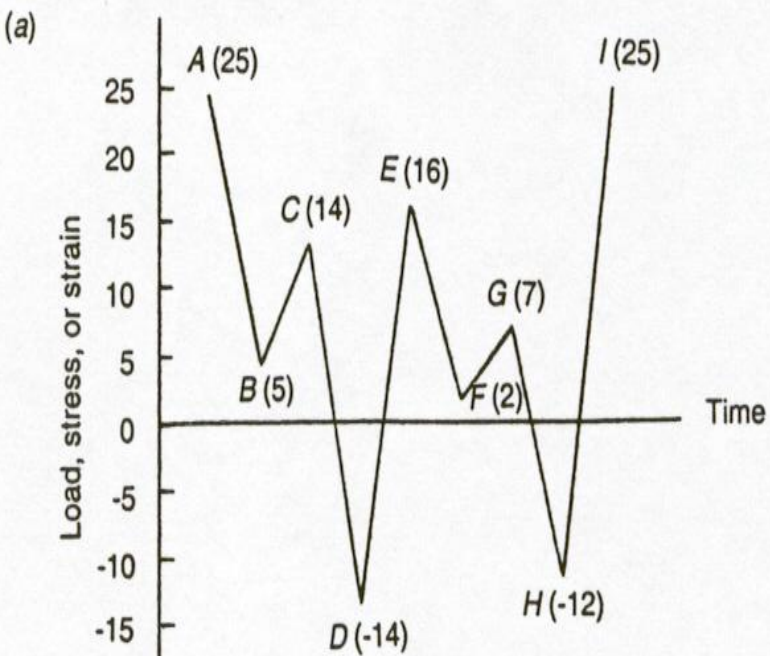
$\sigma_{at\ B} = \sigma_{at\ A} - \Delta\sigma_{A\text{ to }B}$ = 653 – 747 = -94 MPa

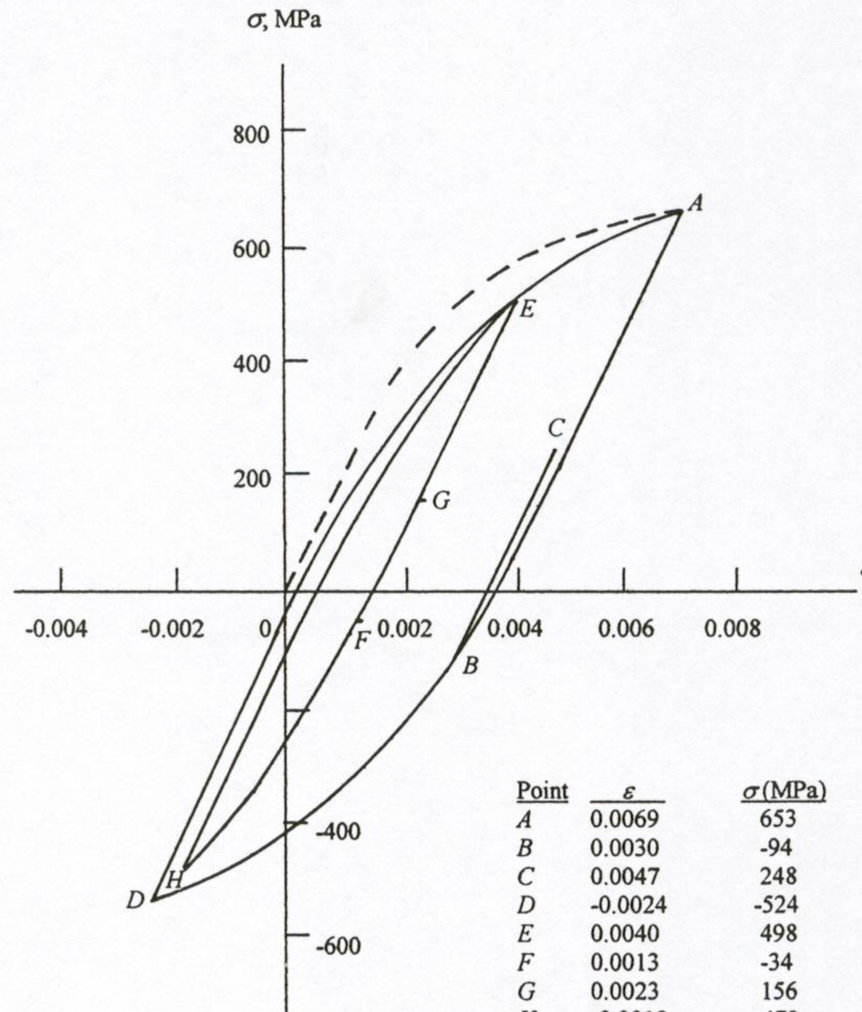
and $\varepsilon_{at\ B} = \varepsilon_{at\ A} - \Delta\varepsilon_{A\text{ to }B}$ = 0.0069 – 0.0039 = 0.0030





LIFE ESTIMATION USING ε-N APPROACH: **EXAMPLE**





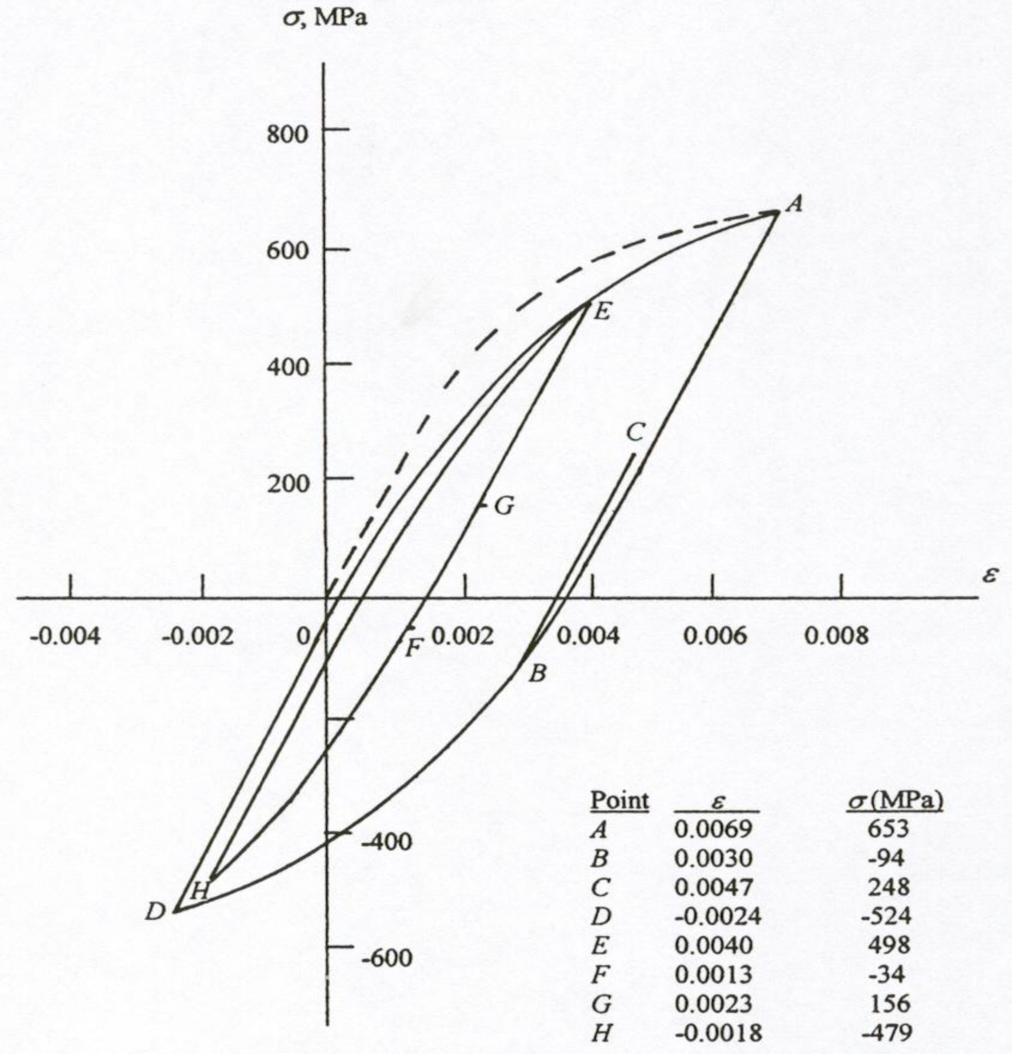
Point	ε	σ (MPa)
A	0.0069	653
B	0.0030	-94
C	0.0047	248
D	-0.0024	-524
E	0.0040	498
F	0.0013	-34
G	0.0023	156
H	-0.0018	-479

LIFE ESTIMATION USING ε-N APPROACH: **EXAMPLE**

- This process is continued throughout the loading, with the hysteresis loop equation remaining the same but the value of the right side of Neuber's equation changing, as ΔS changes for each point in the history.

$$\Delta\varepsilon\,\Delta\sigma = \frac{(K_f\,\Delta S)^2}{E} = \frac{(1.9 \, x \, 400)^2}{200000} = 2.89$$

- **Notch stresses and strains** for the remainder of the loading are summarized and shown in Fig. 9.18.



Point	ε	σ (MPa)
A	0.0069	653
B	0.0030	-94
C	0.0047	248
D	-0.0024	-524
E	0.0040	498
F	0.0013	-34
G	0.0023	156
H	-0.0018	-479

LIFE ESTIMATION USING ε-N APPROACH: **EXAMPLE**

- We can now use the strain-life equation with one of the mean stress correction parameters to calculate **cycles to failure**, N_f, for each combination of strain amplitude and mean stress. Here we choose the Smith-Watson-Topper parameter, where $\sigma_{max} = \sigma_a + \sigma_m$.

$$\varepsilon_a \sigma_{\max} = \frac{(\sigma'_f)^2}{E}(2N_f)^{2b} + \sigma'_f \varepsilon'_f (2N_f)^{b+c} =$$

$$\frac{(1240)^2}{200000}(2N_f)^{2(-0.07)} + (1240)(0.66)(2N_f)^{-0.07-0.69} = 7.69\,(2N_f)^{-0.14} + 818\,(2N_f)^{-.76}$$

LIFE ESTIMATION USING ε-N APPROACH: **EXAMPLE**

- Cycle ratios, n/N_{fi} are then computed and summed, as shown in the following table:

Cycle	ΔS (MPa)	ε_a	σ_{max}	N_f	n	n/N_f
A-D-I	780	0.00465	659	4 050	1	2.47×10^{-4}
B-C-B′	190	0.00085	247	∞	1	0
E-H-E′	560	0.00290	498	112 500	1	9×10^{-6}
F-G-F′	100	0.00050	156	∞	1	0

$$\sum n_i/N_{fi} = 2.56 \times 10^{-4}$$

The **expected fatigue life** is calculated as 1/(2.56x10-4) = 3900 repetitions of the variable amplitude load block.

- If we only use the **largest cycle** for damage calculation (i.e. omitting the other 3 cycles), the expected life is calculated as $1/(2.47 \times 10^{-4})$ = 4050 repetitions. This indicates that **96 percent of the damage** is caused by cycle *A-D-I*.

CRACK GROWTH LIFE ESTIMATION MODELS

- Fatigue crack growth in variable amplitude loading depends not only on the range of stress intensity factor and stress ratio, but very significantly on the previous load history, which may have left compressive or tensile residual stress fields:
  - if compressive, the residual stress field tends to retard (delay) or arrest crack growth;
  - if tensile it tends to accelerate it.

- The major interest in fatigue crack growth life estimations has been motivated by the fail-safe design philosophy in which proper inspection periods must be predetermined.

CRACK GROWTH LIFE ESTIMATION MODELS

- Fatigue crack growth life estimation models for variable amplitude loading can be simple, or quite complex requiring extensive computations.

  - In some cases, the actual service load history can be approximated and simplified to give repeated applications of a multiple block loading sequence as in Fig. 9.2. Estimation of the fatigue crack growth life can then be made evaluating crack growth related to each individual block.

  - Another approach is the summation of crack increments based on each individual cycle.

CRACK GROWTH LIFE ESTIMATION MODELS

- These methods of life estimation can be used assuming that crack growth for a given cycle or block is not influenced by the prior loading history (load sequence), if the load history is highly irregular (random), or if the overloads are not too severe.

- If high overloads can occur and are predominantly in one direction, they may introduce load sequence effects that can significantly affect the fatigue crack growth life. In these cases, models incorporating load sequence effects provide more accurate life estimations.

SIMULATING SERVICE HISTORIES IN LABORATORY

- To avoid the high cost of field failures, **laboratory test methods** have been developed that can approximate the results of field experience for variable amplitude load histories.

- Six different methods in order of increasing physical complexity are:
  (1) characteristic constant amplitude,
  (2) block testing,
  (3) condensed histories,
  (4) truncated histories,
  (5) complete histories, and
  (6) statistically simulated histories.

SIMULATING SERVICE HISTORIES IN THE LABORATORY

- Testing with a **Characteristic Constant Amplitude**.
  - A characteristic constant amplitude, based on experience, serves very well if it can be validated by field data.

  - For automobile suspension springs, for instance, constant amplitude tests to a few hundred thousand cycles of maximum possible deflection reproduce the field history well enough. The reasons for this are:

    (1) the large amplitudes actually do most of the fatigue damage, and

    (2) field use is so diverse that any one field history would be just as wrong as this constant amplitude test.

SIMULATING SERVICE HISTORIES IN THE LABORATORY

- **Block Testing.**
  - The real history is replaced by a number of "blocks" of constant maximum and minimum load or deflection as shown.
  - In principle, the program contains the same number of reversals as the history, and its blocks approximate the distribution of peaks and valleys.
  - In practice large numbers of small ranges, well below the fatigue limit, are omitted.
  - Six to 10 blocks provide adequate approximations.
  - The sequence of blocks is important. A random sequence will minimize undesirable sequence effects.

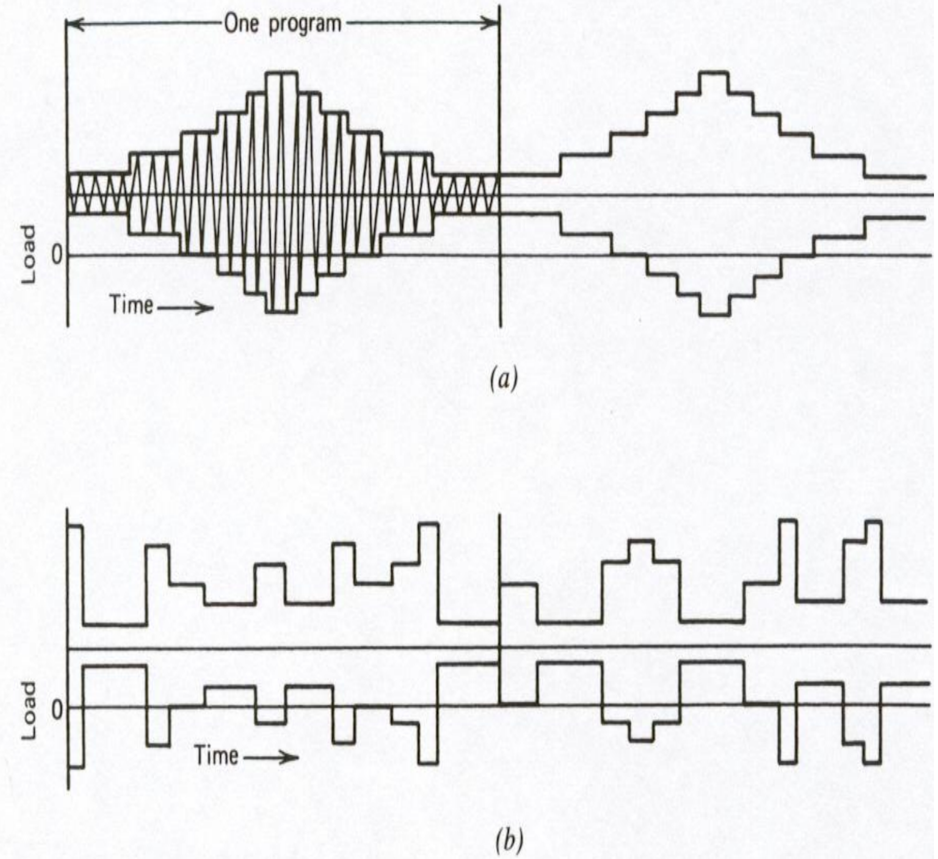


Figure 9.2 Block program load spectra. (*a*) Programmed six load level test. (*b*) Ra dom block program loading.

SIMULATING SERVICE HISTORIES IN THE LABORATORY

- **Testing with Condensed Histories.**
  - To reduce testing cost and time, a condensed version of the real history may be used.
  - Condensed histories include selected peaks and valleys of the real history in their real sequence and omit many peaks and valleys.
  - The selection of the most significant peaks and valleys is done by racetrack counting or by editing a rainflow count to retain only the largest ranges.

- **Truncated Histories.**
  - Histories can be truncated by omitting all ranges smaller than a given amount.
  - If this is done after rainflow counting it amounts to the same as condensed histories.
  - If it is done without regard to overall ranges it may lead to serious errors.

SIMULATING SERVICE HISTORIES IN THE LABORATORY

- **Complete Histories.**
  - Suitable test machines can apply the record of a load history on computer disk or other devices to the test specimen, component, or structure over and over again.
  - This produces a good test, but it may be unnecessarily expensive in time in view of the capabilities of condensed histories.

- **Statistically Simulated Histories.**
  - One can arrange truly random input to the test machine with prescribed parameters, such as the distribution of peaks and valleys or of amplitudes and means.

DIGITAL PROTOTYPING

- Digital prototyping and computer simulation techniques for fatigue analysis are much more recent and rapidly progressing.

- An important element of digital prototyping is dynamic simulation of the structure. This involves:
  - Building a computer model.
  - Exciting the model by representative service loads, such as a digital road profile for an automobile.
  - The output from the excited computer model can then be used for fatigue damage assessment and life predictions.

DIGITAL PROTOTYPING

- Building the computer or digital model consists of
  - representing the structure by multibody elements (such as a control arm of an automobile represented by rigid or flexible mass elements),
  - connecting them by appropriate force elements (such as springs or damping elements) or appropriate constraints (such as spherical joints).

- In building a model, there is always a trade-off between a complex and expensive model with better solution accuracy, versus a simpler less expensive model but lower solution accuracy.

DIGITAL PROTOTYPING

- Commercial software including both multibody dynamic and finite element analyses are now commonly used for dynamic simulation.
  - The multibody dynamic analysis software is generally used to obtain the dynamic response of the structure, such as displacements and reaction forces.
  - The dynamic response is then used as the input to the finite element analysis software to obtain stresses and strains at the critical locations
  - These in turn are used for fatigue life predictions.

SUMMARY AND DOS AND DON'TS IN DESIGN

- Service load histories are usually variable amplitude and their **realistic representation** is a key ingredient to successful fatigue design. **Transducers** in the critical locations of the component or structure are usually used to measure the load or strain history.

- To compare fatigue behavior from variable amplitude histories to fatigue curves obtained with simple constant amplitude loading, a cycle counting method is needed. **Rainflow method** is the most popular method.

- Damage from each cycle is accumulated or summed over the entire load history by using a cumulative damage rule. The **linear damage rule**, also referred to as the Palmgren-Miner rule, is the simplest and most common rule, $\sum n_i / N_{fi} = 1$.

SUMMARY AND **DOS AND DON'TS** IN DESIGN

- Do ask yourself whether **sequence effects** are likely to be important. Infrequent one-sided overloads are expected to produce sequence effects. For many service histories, however, sequence effects either cancel each other or are entirely unpredictable.

- Don't ignore the fact that often only a few events in the load history produce most of the damage. Properly **condensed load histories** containing these events can significantly reduce analysis as well as testing time.

- Don't forget that only a few overloads in the load history can significantly affect the fatigue behavior in a notched or cracked component by producing beneficial compressive or detrimental tensile residual stresses at the notch root or at the crack tip.

SUMMARY AND DOS AND DON'TS IN DESIGN

- The **stress-life approach** neglects load sequence effects in service load histories. However, it is simple and the *S-N* curve can include many of the complex factors influencing fatigue behavior.

- The **strain-life approach** accounts for load sequence effects and is generally an advantageous approach for cumulative damage analysis of notch members, where significant plasticity usually exists.

- Both the *S-N* and ε-*N* approaches are used to predict life to **crack formation or nucleation**, with a crack length on the order of 1 mm.

SUMMARY AND DOS AND DON'TS IN DESIGN

- Fatigue **crack growth life** estimations for variable amplitude loading can be made by either neglecting or taking into account load sequence effects. Current fatigue crack growth life estimation codes that account for load sequence effects use crack tip plasticity or crack closure models.

- To avoid the high cost of field failures, **laboratory and/or field test methods** may still be required to complement fatigue analysis and life predictions from variable amplitude load histories. Laboratory and/or field test methods with different degrees of physical complexity and cost can be used to approximate the results of actual service experience.