CYCLIC DEFORMATION & STRAIN-LIFE (ε -*N*) APPROACH

- MONOTONIC TENSION TEST AND STRESS-STRAIN BEHAVIOR
- STRAIN-CONTROLLED TEST METHODS
- CYCLIC DEFORMATION AND STRESS-STRAIN BEHAVIOR
- STRAIN-BASED APPROACH TO LIFE ESTIMATION, ε-*N*
  - DETERMINATION OF STRAIN-LIFE FATIGUE PROPERTIES
  - MEAN STRESS EFFECTS
  - FACTORS INFLUENCING STRAIN-LIFE BEHAVIOR

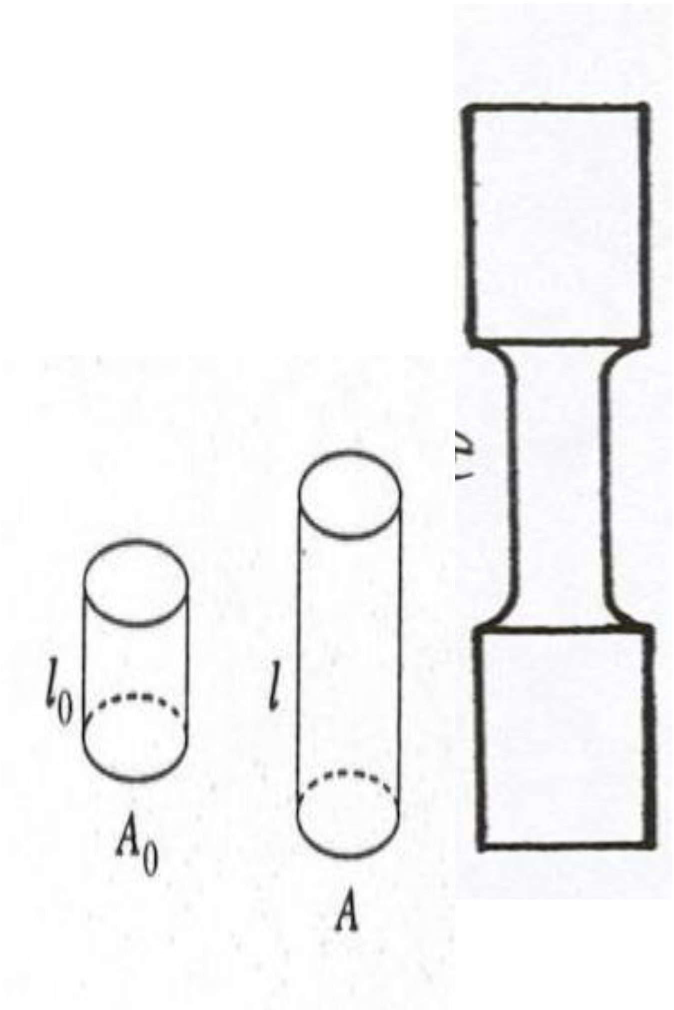
MONOTONIC TENSION TEST
AND
STRESS-STRAIN BEHAVIOR

MONOTONIC TENSION TEST AND STRESS-STRAIN BEHAVIOR

- Monotonic tension stress-strain properties are usually reported in handbooks and are used in many specifications.
- Monotonic behavior is obtained from a *tension test* where a specimen with circular or rectangular cross section within the uniform gage length is subjected to a monotonically increasing tensile force until it fractures.
- They are *easy tests to perform* and provide information that has become conventionally accepted.
- However, their relation to fatigue behavior may be *remote*.

MONOTONIC TENSION TEST AND STRESS-STRAIN BEHAVIOR

- Details of tension testing for metallic materials are provided in ASTM standard E8 or E8M.
- Monotonic uniaxial stress-strain behavior can be based on "**engineering**" stress-strain or "**true**" stress-strain relationships.
- The difference is in using *original versus instantaneous* gage section dimensions.



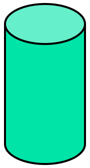
MONOTONIC TENSION TEST AND STRESS-STRAIN BEHAVIOR

- The nominal **engineering stress**, *S*, in a uniaxial test specimen is defined by:

P = axial force

A_o = original cross sectional area

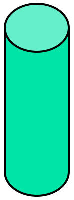
$$S = \frac{P}{A_o}$$



- The **true stress**, σ, is given by:

A = instantaneous cross-sectional area

$$\sigma = \frac{P}{A}$$



The true stress in tension is larger than the engineering stress since the cross-sectional area decreases during loading.

MONOTONIC TENSION TEST AND STRESS-STRAIN BEHAVIOR

- The **engineering strain**, *e*, is based on the original gage length and is given by:

$$e = \frac{(l - l_o)}{l_o} = \frac{\Delta l}{l_o}$$

l = instantaneous gage length

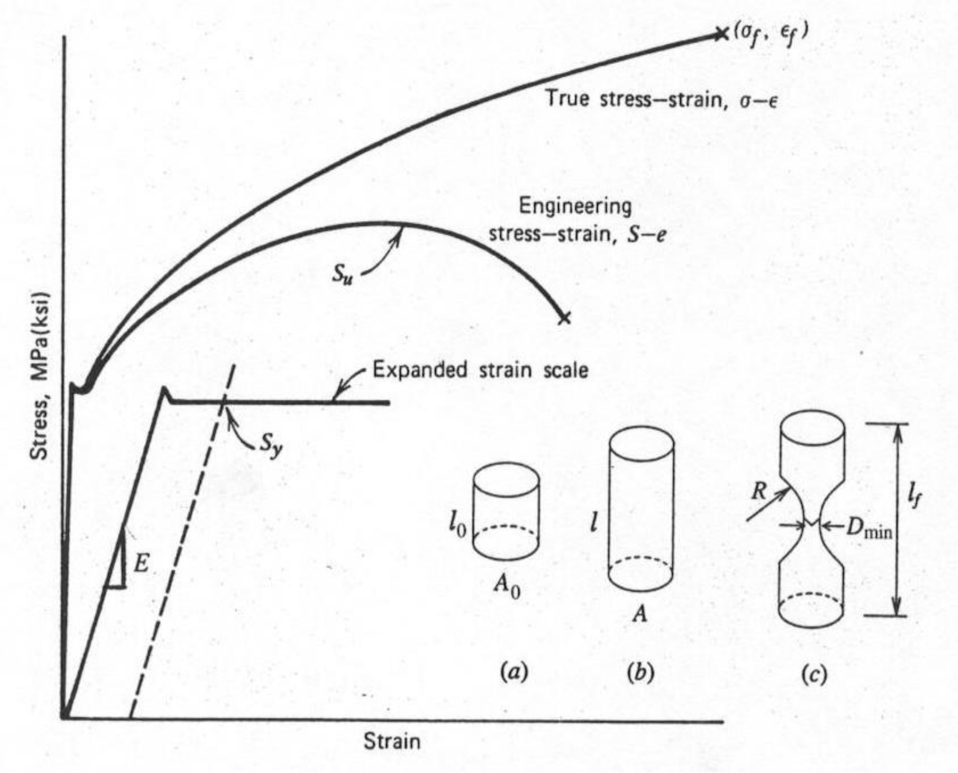
Δl = the change in length of the original gage length lo.

- The **true** or **natural strain**, ε, is based on the instantaneous gage length and is given by:

$$d\varepsilon = \frac{dl}{l} \quad \text{or} \quad \varepsilon = \int \frac{dl}{l} = \ln\left(\frac{l}{l_o}\right)$$

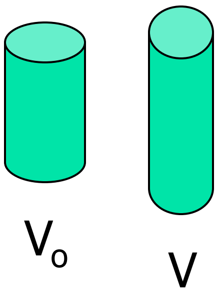
MONOTONIC TENSION TEST AND STRESS-STRAIN BEHAVIOR

- For small strains, less than about 2 percent, the "engineering" stress, *S*, is approximately equal to the "true" stress, σ, and the "engineering" strain, *e*, is approximately equal to the "true" strain, ε.
- No distinction between "engineering" and "true" components is needed for these small strains.
- For larger strains the differences become appreciable.



MONOTONIC TENSION TEST AND STRESS-STRAIN BEHAVIOR

- A constant volume condition can be assumed such that up to necking: $A\,l = A_o\,l_o$.



- The following relationships can then be derived.

$$\sigma = S(1+e) \qquad \varepsilon = \ln\frac{A_o}{A} = \ln(1+e)$$

- These equations are valid up to necking which takes place when the ultimate strength is reached.

- After necking plastic deformation becomes localized and strain is no longer uniform throughout the gage section.

Properties obtained from monotonic tensile tests

E = modulus of elasticity, MPa

S_y = yield strength, MPa

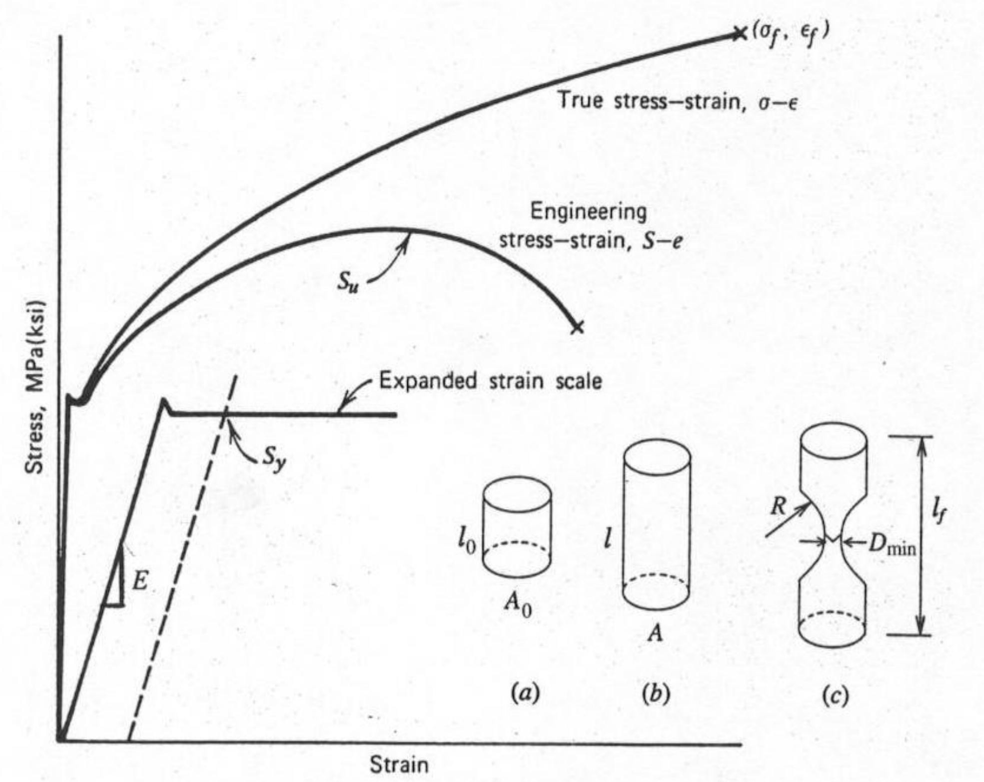
S_u = ultimate tensile strength, MPa
= P_{max}/Ao

σ_f = true fracture strength, MPa

%*RA* = percent reduction in area
= 100 $(Ao - A_f)/Ao$

ε_f = true fracture strain or ductility = ln (Ao/A_f) = ln [100/(100 - %*RA*)]

%*EL* = percent elongation
= 100 $(l_f - lo)/lo$

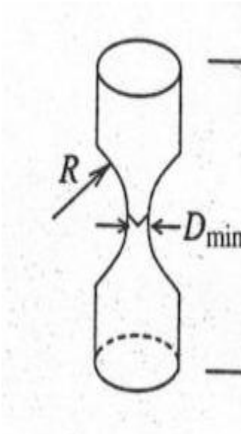


MONOTONIC TENSION TEST AND STRESS-STRAIN BEHAVIOR

- The **true fracture strength**, σ_f, can be calculated from P_f/A_f.

- It is usually corrected for *necking*, which causes a biaxial state of stress at the neck surface and a triaxial state of stress at the neck interior.

- The **Bridgman correction factor** is used to compensate for triaxial state of stress and applies to cylindrical specimens.



$$\sigma_f = \frac{P_f/A_f}{\left(1 + 4R/D_{\min}\right)\ \ln\left(1 + D_{\min}/4R\right)}$$

R = the radius of curvature of the neck

D_{min} = the diameter of the cross-section in the thinnest part of the neck.

- Materials with a brittle tensile behavior do not exhibit necking, and therefore, do not require this correction factor.

MONOTONIC TENSION TEST AND STRESS-STRAIN BEHAVIOR

- Values of S_y, S_u, and σ_f, are indicators of material **strength**.
- Values of $\%RA$, $\%EL$, and ε_f are indicators of material **ductility**.
- Representative values of monotonic tensile material properties are given in Tables A.1 and A.2 for selected engineering alloys.

Material	Process Description	Hardness	S_u MPa(ksi)	S_y MPa(ksi)	%El	%RA	S_f[a] MPa(ksi)
Steels[b]	*S_f based on 10^6 to 10^8 cycles to failure*						
1020	Annealed	111HB	393(57)	296(43)	36	66	138(20)
1020	Hot Rolled	143HB	448(65)	331(65)	36	59	241(35)
1040	Annealed	149HB	517(75)	351(51)	30	57	269(39)
1050	Annealed	187HB	634(92)	365(53)	24	40	365(53)
4130	Normalized	197HB	668(97)	434(63)	26	60	324(47)
4130	WQ&T 650C	245HB	809(118)	703(102)	22	64	489(71)
4140	OQ&T 650C	285HB	758(110)	655(95)	18	53	420(61)
4140	OQ&T 540C	358HB	1137(165)	985(143)	15	50	455(66)
4340	OQ&T 540C	380HB	1261(183)	1171(170)	14	52	668(97)
4340	OQ&T 425C	430HB	1530(222)	1378(200)	12	47	468(68)
5140	OQ&T 540C	311HB	1068(155)	923(134)	17	53	620(90)
5140	OQ&T 425C	375HB	1309(190)	1164(169)	12	42	565(82)
8640	OQ&T 540C	331HB	1068(155)	944(137)	17	56	537(78)
HY-140	Q&T 540C	34Rc	1027(149)	978(142)	20	65	482(70)
H-11	Q&T	52Rc	1791(260)	1447(210)	8	52	634(92)
300M	Q&T 260C	52Rc	1791(260)	1585(230)	12	37	620(90)
D6AC	Q&T 260C	54Rc	1998(290)	1722(250)	9	36	689(100)
9Ni-4Co-25	T 540C	36Rc	1378(200)	1309(190)	17	70	758(110)
9Ni-4Co-45	T 315C	48Rc	1929(280)	1757(255)	5	35	612(90)
18Ni 200 marage	Aged 480C	43Rc	1550(225)	1481(215)	11	55	689(100)
18Ni 250 marage	VM Aged 480C	50Rc	1764(256)	1633(237)	11	62	689(100)
18Ni 300 marage	VM Aged 480C	55Rc	1984(288)	1922(279)	7	50	758(110)
18Ni 350 marage	VM Aged 480C	59Rc	2425(352)	2377(345)	8	44	758(110)
302	Annealed	80Rb	640(93)	276(40)	68	65	234(34)
302	CR 40%	35Rc	1040(151)	909(132)	13	8	517(75).
304	Annealed	80Rb	599(87)	234(34)	57	67	241(35)
304	CW 10%	10Rc	675(98)	482(70)	35		413(60)
304	CW 40%	35Rc	1006(146)	930(135)	12		634(92)
316	Annealed	77Rb	586(85)	262(38)	61	67	269(39)
403	Annealed	155HB	517(75)	310(45)	30	70	276(40)
403	T 650C	97Rb	758(110)	585(85)	23	65	379(55)
Aluminum Alloys[c]	*S_f based on $5x10^8$ cycles to failure*						
1100-0	Annealed	23HB	90(13)	35(5)	45		35(5)
2014-T6	Sol. Treat Aged	135HB	482(70)	413(60)	13		124(18)
2024-T3	Sol. Treat CW Aged	120HB	482(70)	345(50)	18		138(20)
2024-T4	Sol. Treat Aged	120HB	468(68)	324(47)	19		138(20)
2219-T851	Sol. Treat CW Aged		455(66)	351(51)	10		103(15)
3003-H16	Strain Hardened	47HB	179(26)	172(25)	14		69(10)
3004-H36	Strain Hardened	70HB	262(38)	227(33)	9		110(16)
6061-T4	Sol. Treat Aged	65HB	242(35)	145(21)	25		96(14)
7075-T6	Sol. Treat Aged	150 HB	572(83)	503(73)	11		158(23)
Others							
Ti[d]	Annealed		520(75)				330(52)
Ti-6Al-4V[e]			1190(172)	1090(158)			365(53)
Copper[d]	Annealed		235(34)	75(11)			75(11)
60/40 Brass[d]	Annealed		315(46)	95(14)			85(12)
Phospher Bronze[d]	Annealed		340(49)	150(22)			170(25)

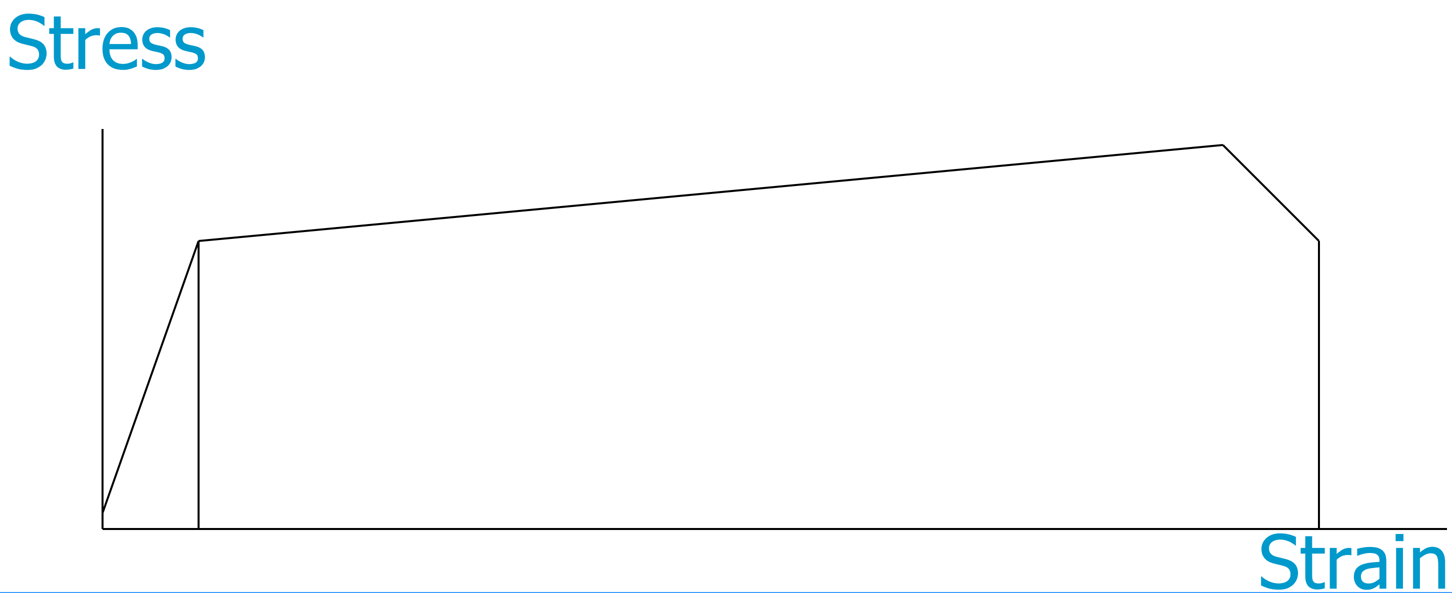
[a] Incomplete information on surface finish. These values *do not* represent design fatigue limits.

Table A.2 Monotonic, Cyclic, and Strain-life Properties of Selected Engineering Alloys[a,b,c]

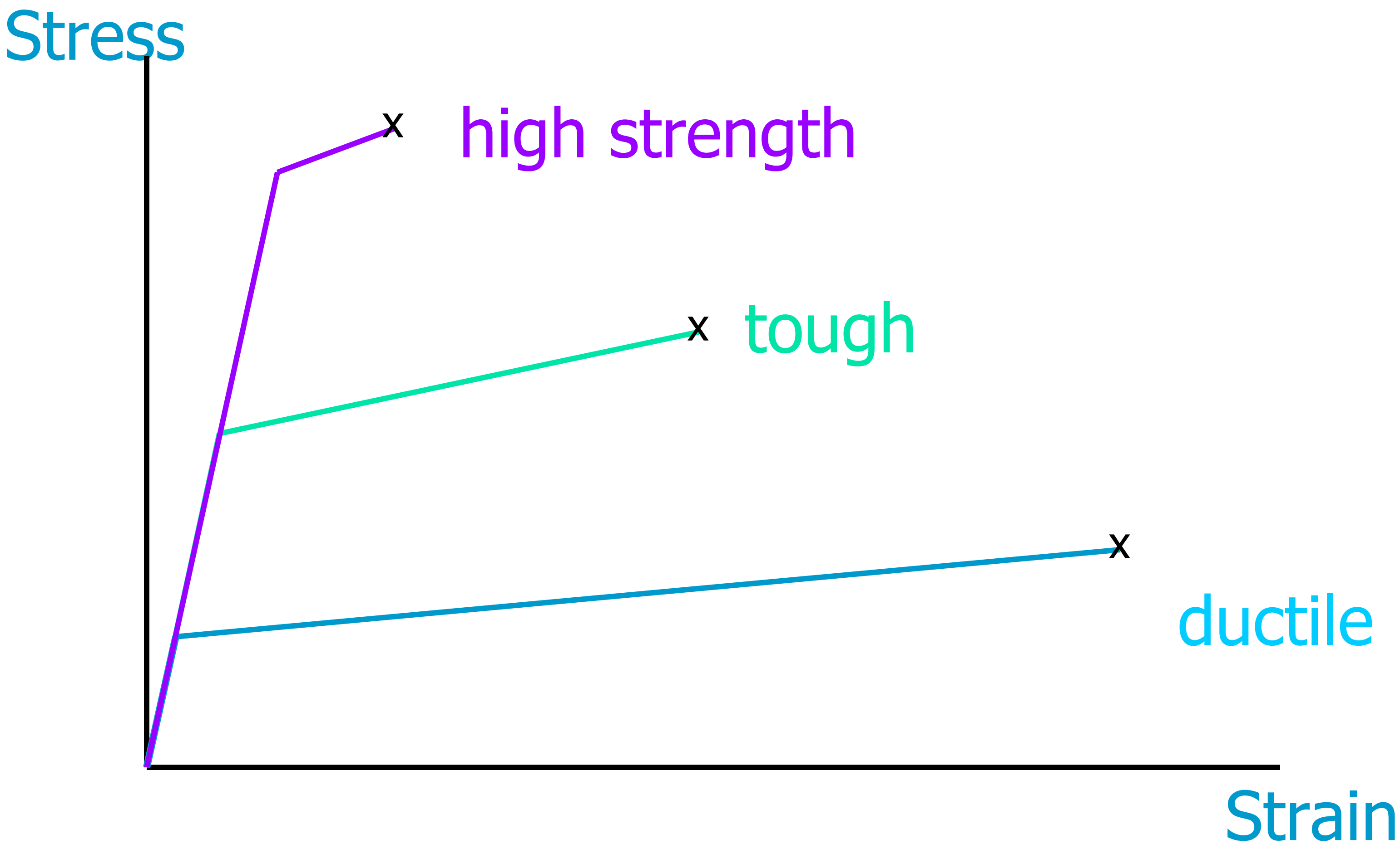
Material	Process Description	S_u MPa (ksi)	HB	E GPa (ksi $\cdot 10^3$)	%RA	S_y/S_y' MPa (ksi)	K/K' MPa (ksi)	n/n'	$\varepsilon_f/\varepsilon_f'$	σ_f/σ_f' MPa (ksi)	b	c
Steel												
1010	HR sheet	331	—	203	80	200/—	534/867	0.185/0.244	1.63/0.104	—/499	-0.100	-0.408
		(48)		(29.5)		(29)/ —	(78)/(126)			—/(72)		
1020	HR sheet	441	109	203	62	262/—	738/1962	0.190/0.321	0.96/0.337	—/1384	-0.156	-0.485
		(64)		(29.5)		(38)/ —	(107)/(284)			—/(201)		
1038[c]	Normalized	582	163	201	54	331/342	1106/1340	0.259/0.220	0.77/0.309	898/1043	-0.107	-0.481
		(84)		(29.5)		(48)/(50)	(160)/(195)			(130)/(151)		
1038[c]	Q & T	649	195	219	67	410/364	1183/1330	0.221/0.208	1.10/0.255	1197/1009	-0.097	-0.460
		(94)		(31.5)		(60)/(53)	(172)/(193)			(174)/(146)		
Man-Ten	HR sheet	510	—	207	64	393/372	—/786	0.20/0.11	1.02/0.86	814/807	-0.071	-0.65
		(74)		(30)		(57)/(54)	—/(114)			(118)/(117)		
RQC-100	HR sheet	931	290	207	64	883/600	1172/1434	0.06/0.14	1.02/0.66	1330/1240	-0.07	-0.69
		(135)		(30)		(128)/(87)	(170)/(208)			(193)/(180)		
1045	Annealed	752	225	—	44	517/—	—/1022	—/0.152	0.58/0.486	—/916	-0.079	-0.520
		(109)		—		(75)/—	—/(148)			—/(133)		
1045	Q & T	1827	500	207	51	1689/—	—/3371	0.047/0.145	0.71/0.196	—/2661	-0.093	-0.643
		(265)		(30)		(245)/ —	—/(489)			—/(386)		
1090[c]	Normalized	1090	259	203	14	735/545	1765/1611	0.158/0.174	0.15/0.250	—/1310	-0.091	-0.496
		(158)		(29.5)		(107)/(79)	(256)/(234)			—/(190)		
1090[c]	Q & T	1147	309	217	22	650/627	1895/1873	0.165/0.176	0.24/0.700	—/1878	-0.120	-0.600
		(166)		(31.5)		(94)/(91)	(275)/(272)			—/(273)		
1141[c]	Normalized	789	229	220	47	493/481	1379/1441	0.187/0.177	0.64/0.602	1117/1326	-0.103	-0.581
		(115)		(32)		(72)/(70)	(200)/(209)			(162)/(192)		
1141[c]	Q & T	925	277	227	59	814/591	1205/1277	0.074/0.124	0.88/0.309	1405/1127	-0.066	-0.514
		(134)		(33)		(118)/(86)	(125)/(185)			(204)/(164)		
4142	Q & T	1413	380	207	48	1378/—	—/2266	0.051/0.124	0.65/0.637	—/2143	-0.094	-0.761
		(205)		(30)		(200)/—	—/(387)			—/(311)		
4142	Q & T	1929	475	207	35	1722/—	—/2399	0.048/0.094	0.43/0.331	—/2161	-0.081	-0.854
		(280)		(30)		(250)/—	—/(348)			—/(314)		
4340	HR	827	243	193	43	634/—	—/1337	—/0.168	0.57/0.522	—/1198	-0.095	-0.563
		(120)		(28)		(92)/—	—/(194)			—/(174)		

MONOTONIC TENSION TEST AND STRESS-STRAIN BEHAVIOR

- Indicators of **energy absorption capacity** of a material are resilience and tensile toughness.
  - **Resilience** is the elastic energy absorbed by the specimen and is equal to the area under the elastic portion of the stress-strain curve.
  - **Tensile toughness** is the total energy density or energy per unit volume absorbed during deformation (up to fracture) and is equal to the total area under the engineering stress-strain curve.



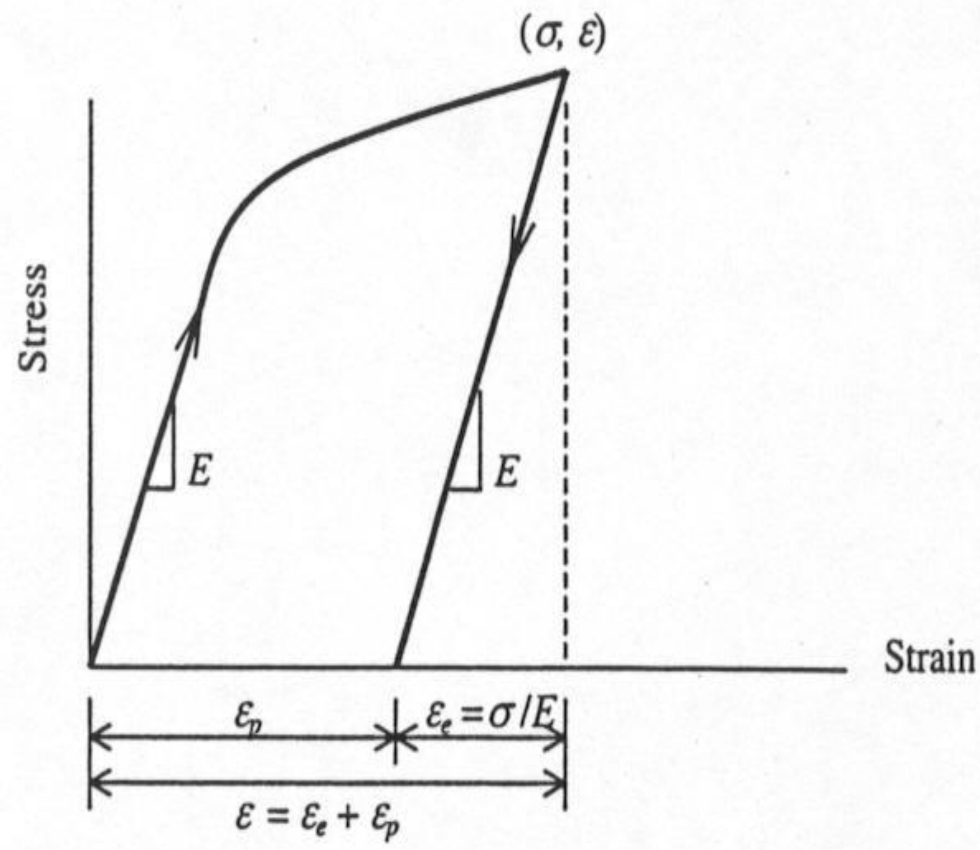
A material with high tensile toughness with a good combination of both high strength and ductility is often desirable.



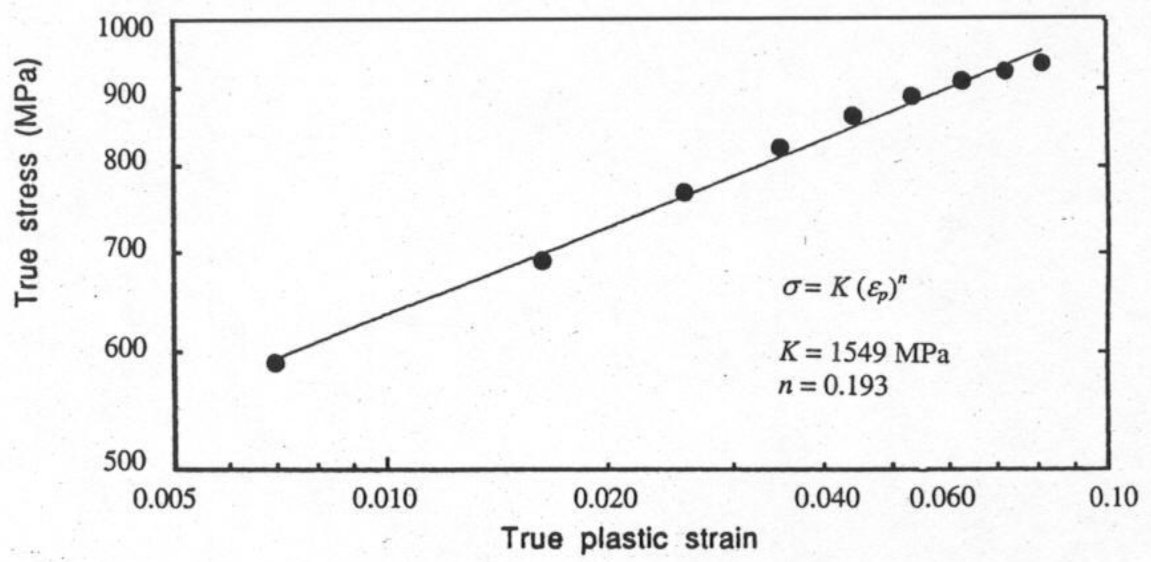
- Inelastic or **plastic strain** results in permanent deformation which is not recovered upon unloading.

- The unloading curve is elastic and parallel to the initial elastic loading line.

- The total strain, ε, is composed of two components,
  - an elastic strain, $\varepsilon_e = \sigma / E$, and
  - a plastic component, ε_p.



- For many metals, a **plot of true stress versus true plastic strain** in log-log coordinates results in a linear curve.
- An example of such a plot is shown for AISI 11V41 steel.
- To avoid necking influence, only data between the yield strength and ultimate strength portions of the stress-strain curve are used to generate this plot.
- This curve is represented by the power function: $\sigma = K(\varepsilon_p)^n$



$$\sigma = K(\varepsilon_p)^n$$

K is the strength coefficient (stress intercept at $\varepsilon_p = 1$)

n is the strain hardening exponent (slope of the line).

- *The total true strain is given by:*

$$\varepsilon = \varepsilon_e + \varepsilon_p = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{1/n}$$

  - This type of true stress-true strain relationship is often referred to as the ***Ramberg-Osgood relationship***.
  - Value of **n** gives a measure of the material's work hardening behavior.
  - K and n for some engineering alloys are also given in Table A.2.

Table A.2 Monotonic, Cyclic, and Strain-life Properties of Selected Engineering Alloys[a,b,c]

Material	Process Description	S_u MPa (ksi)	HB	E GPa (ksi $\cdot 10^3$)	%RA	S_y/S_y' MPa (ksi)	K/K' MPa (ksi)	n/n'	$\varepsilon_f/\varepsilon_f'$	σ_f/σ_f' MPa (ksi)	b	c
Steel												
1010	HR sheet	331	—	203	80	200/—	534/867	0.185/0.244	1.63/0.104	—/499	-0.100	-0.408
		(48)		(29.5)		(29)/ —	(78)/(126)			—/(72)		
1020	HR sheet	441	109	203	62	262/—	738/1962	0.190/0.321	0.96/0.337	—/1384	-0.156	-0.485
		(64)		(29.5)		(38)/ —	(107)/(284)			—/(201)		
1038[c]	Normalized	582	163	201	54	331/342	1106/1340	0.259/0.220	0.77/0.309	898/1043	-0.107	-0.481
		(84)		(29.5)		(48)/(50)	(160)/(195)			(130)/(151)		
1038[c]	Q & T	649	195	219	67	410/364	1183/1330	0.221/0.208	1.10/0.255	1197/1009	-0.097	-0.460
		(94)		(31.5)		(60)/(53)	(172)/(193)			(174)/(146)		
Man-Ten	HR sheet	510	—	207	64	393/372	—/786	0.20/0.11	1.02/0.86	814/807	-0.071	-0.65
		(74)		(30)		(57)/(54)	—/(114)			(118)/(117)		
RQC-100	HR sheet	931	290	207	64	883/600	1172/1434	0.06/0.14	1.02/0.66	1330/1240	-0.07	-0.69
		(135)		(30)		(128)/(87)	(170)/(208)			(193)/(180)		
1045	Annealed	752	225	—	44	517/—	—/1022	—/0.152	0.58/0.486	—/916	-0.079	-0.520
		(109)		—		(75)/—	—/(148)			—/(133)		
1045	Q & T	1827	500	207	51	1689/—	—/3371	0.047/0.145	0.71/0.196	—/2661	-0.093	-0.643
		(265)		(30)		(245)/ —	—/(489)			—/(386)		
1090[c]	Normalized	1090	259	203	14	735/545	1765/1611	0.158/0.174	0.15/0.250	—/1310	-0.091	-0.496
		(158)		(29.5)		(107)/(79)	(256)/(234)			—/(190)		
1090[c]	Q & T	1147	309	217	22	650/627	1895/1873	0.165/0.176	0.24/0.700	—/1878	-0.120	-0.600
		(166)		(31.5)		(94)/(91)	(275)/(272)			—/(273)		
1141[c]	Normalized	789	229	220	47	493/481	1379/1441	0.187/0.177	0.64/0.602	1117/1326	-0.103	-0.581
		(115)		(32)		(72)/(70)	(200)/(209)			(162)/(192)		
1141[c]	Q & T	925	277	227	59	814/591	1205/1277	0.074/0.124	0.88/0.309	1405/1127	-0.066	-0.514
		(134)		(33)		(118)/(86)	(125)/(185)			(204)/(164)		
4142	Q & T	1413	380	207	48	1378/—	—/2266	0.051/0.124	0.65/0.637	—/2143	-0.094	-0.761
		(205)		(30)		(200)/—	—/(387)			—/(311)		
4142	Q & T	1929	475	207	35	1722/—	—/2399	0.048/0.094	0.43/0.331	—/2161	-0.081	-0.854
		(280)		(30)		(250)/—	—/(348)			—/(314)		
4340	HR	827	243	193	43	634/—	—/1337	—/0.168	0.57/0.522	—/1198	-0.095	-0.563
		(120)		(28)		(92)/—	—/(194)			—/(174)		

MONOTONIC TENSION TEST AND STRESS-STRAIN BEHAVIOR

- Stress-strain behavior of a material can be sensitive to the **strain rate**, particularly at elevated temperatures.

- A significant increase in the strain rate generally increases strength but reduces ductility of the material.

- For metals and alloys, however, the strain rate effect can be small at room temperature.

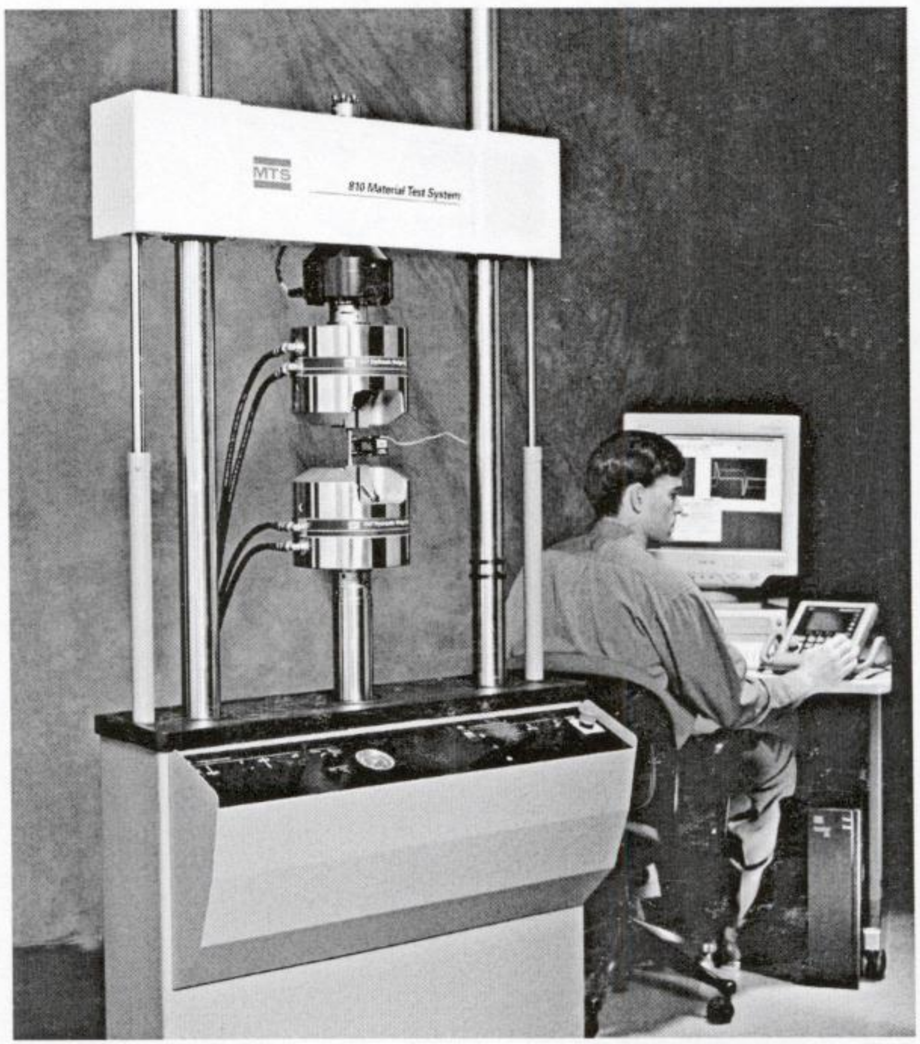
STRAIN-CONTROLLED TEST METHODS

STRAIN-CONTROLLED TEST METHODS

- An important aspect of the fatigue process is **plastic deformation**. Fatigue cracks usually nucleate from plastic straining in localized regions.

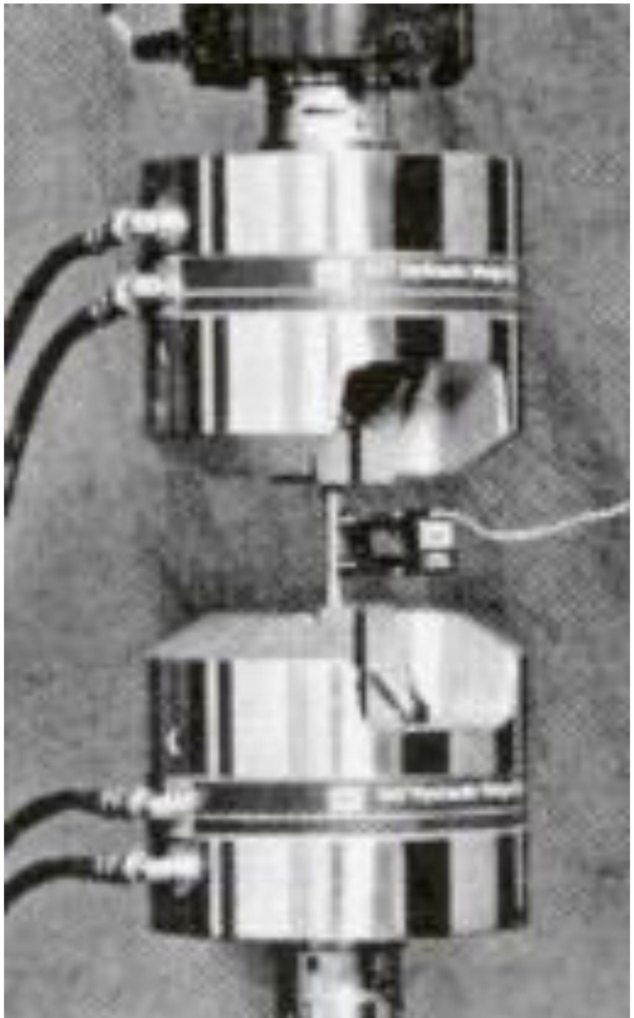
- Therefore, cyclic **strain-controlled tests** can better characterize fatigue behavior of a material than cyclic stress-controlled tests, particularly in the low cycle fatigue region and/or in notched members.

- Strain-controlled fatigue testing has become very common, even though the testing equipment and control are more complicated than the traditional load or stress-controlled testing.

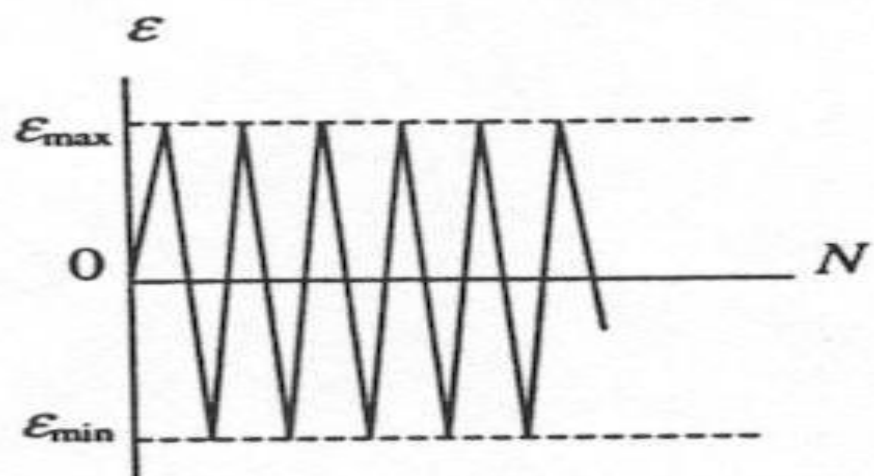

MTS
810 Material Test System

STRAIN-CONTROLLED TEST METHODS

- Strain-controlled testing is usually conducted on a servo-controlled closed-loop testing machine.
  - A *uniform gage section* smooth specimen is subjected to axial straining.
  - An **extensometer** is attached to the uniform gage length to control and measure strain over the gage section.
  - A standard strain-controlled test consists of *constant amplitude completely reversed straining* at a constant or nearly constant strain rate.
  - The most common strain-time control signals used are triangular (sawtooth) and sinusoidal *waveforms*.
  - Stress response generally changes with continued cycling. *Stress and plastic strain variations* are usually recorded periodically throughout the test and cycling is continued until fatigue failure occurs.
  - **ASTM *Standard E606***: Strain-Controlled Fatigue Testing.




Epsilon
TECHNOLOGY CORP

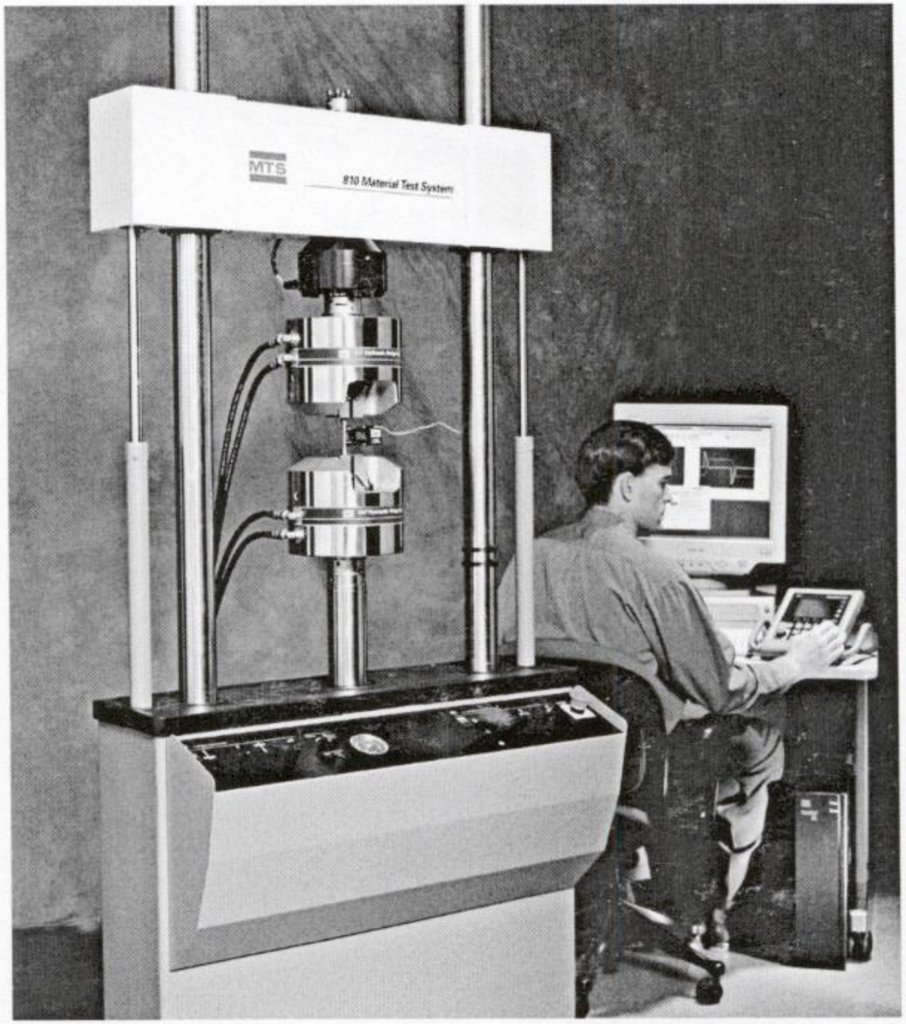

ε
εmax
O
N
εmin

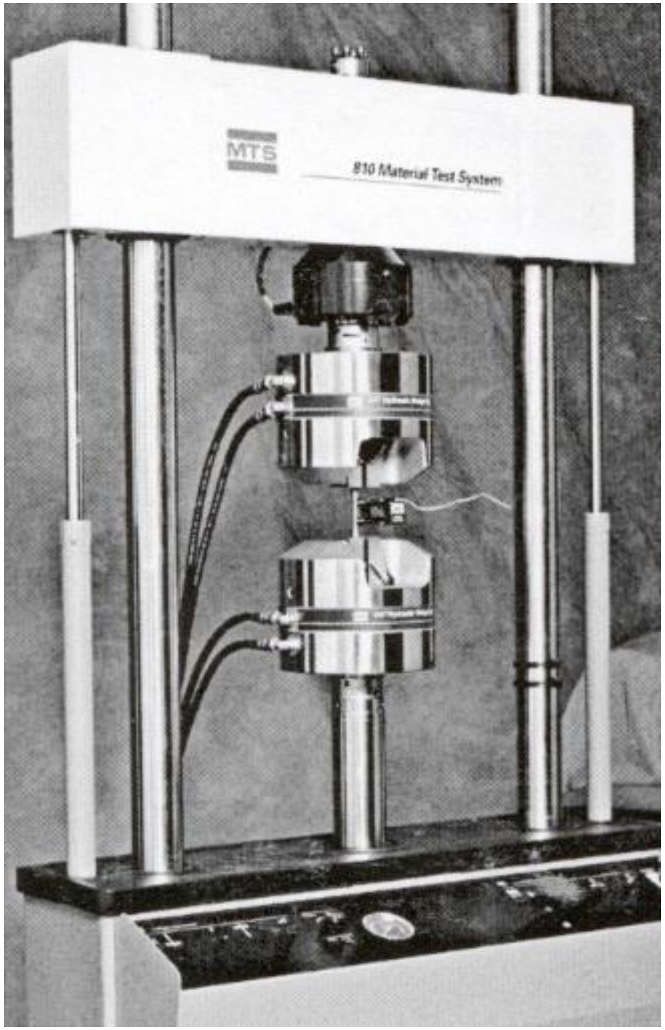
STRAIN-CONTROLLED TEST METHODS

- An important consideration in axial fatigue testing is *uniformity of stress and strains* in the specimen gage section.

- A major source of non-uniformity of gage section stress and strains is a bending moment resulting from **specimen misalignment** that can significantly shorten the fatigue life. Specimen misalignment can result from:

    - eccentricity and/or tilt in the load-train components (including load cell, grips, and load actuator),
    - improper specimen gripping,
    - lateral movement of the load-train components during the test due to their inadequate stiffness.


MTS
810 Material Test System


MTS
810 Material Test System

CYCLIC DEFORMATION AND STRESS-STRAIN BEHAVIOR

CYCLIC DEFORMATION AND STRESS-STRAIN BEHAVIOR

- The stress-strain behavior obtained from a monotonic test can be quite different from that obtained under cyclic loading.

- This was first observed by **Bauschinger**. His experiments indicated the yield strength in tension or compression was reduced after applying a load of the opposite sign that caused inelastic deformation.

- Thus, one single reversal of inelastic strain can change the stress-strain behavior of metals.

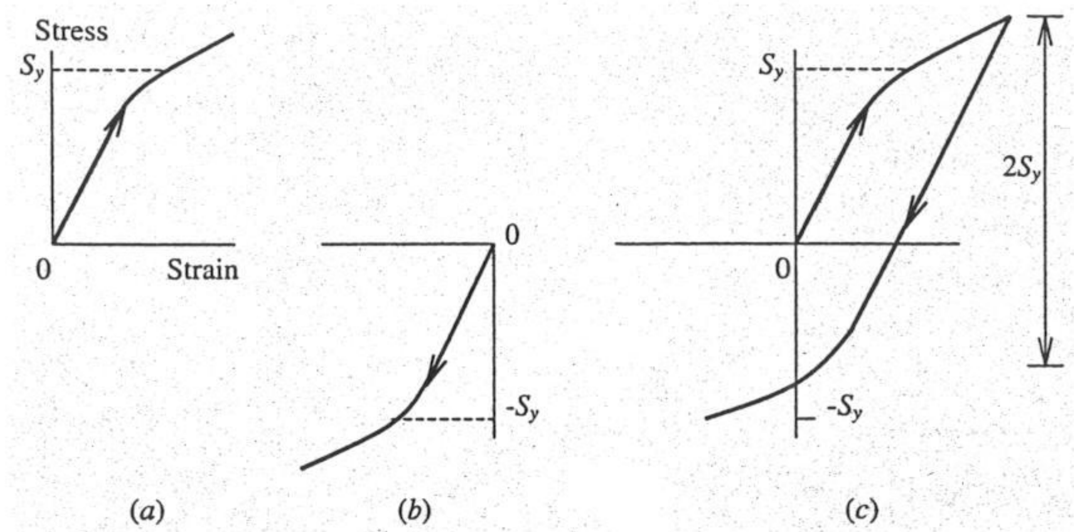
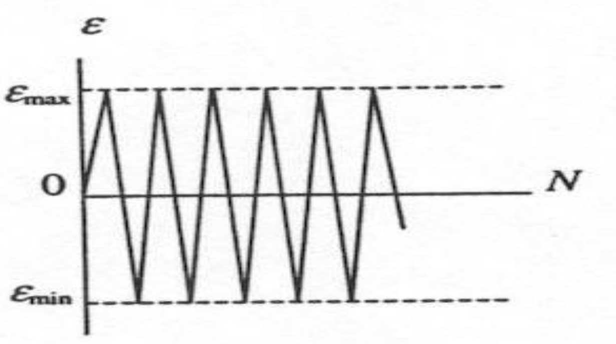
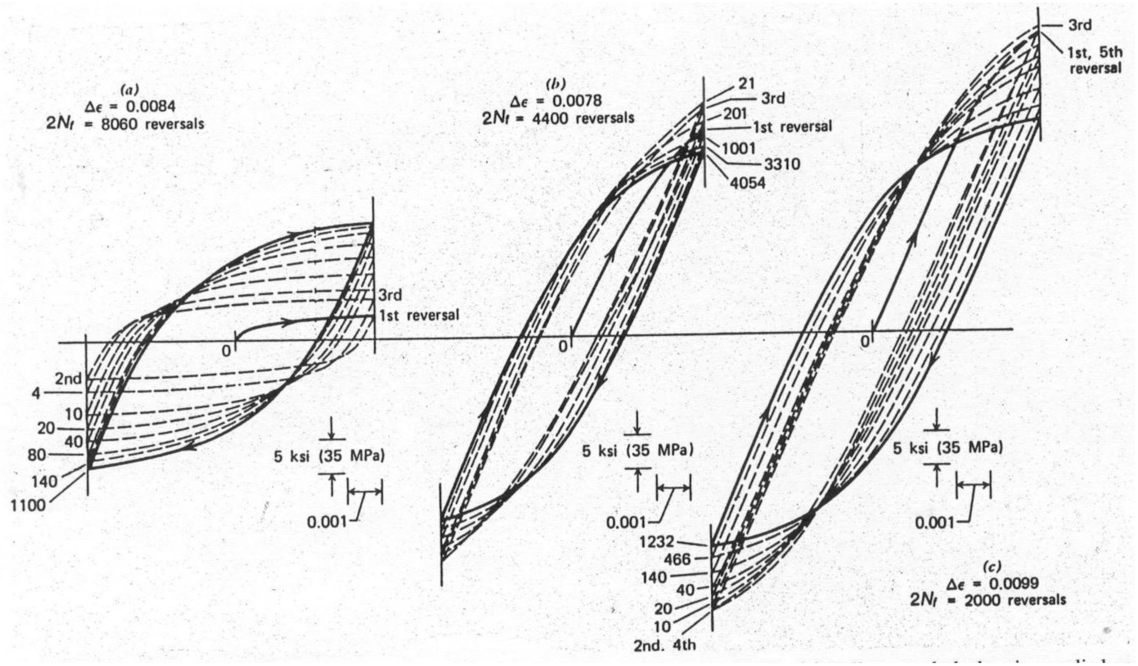


Figure 5.4 Bauschinger effect. (*a*) Tension loading. (*b*) Compression loading. (*c*) Tension loading followed by compression loading.

CYCLIC DEFORMATION AND STRESS-STRAIN BEHAVIOR

- Stress-strain curves during **cyclic strain-controlled testing of copper**:
  - (a) fully annealed condition,
  - (b) partially annealed condition, and
  - (c) cold-worked condition.
- The area within a hysteresis loop is energy dissipated during a cycle (usually in the form of heating). This energy represents the plastic work from the cycle.
- Appreciable progressive change in stress-strain behavior during inelastic cycling.





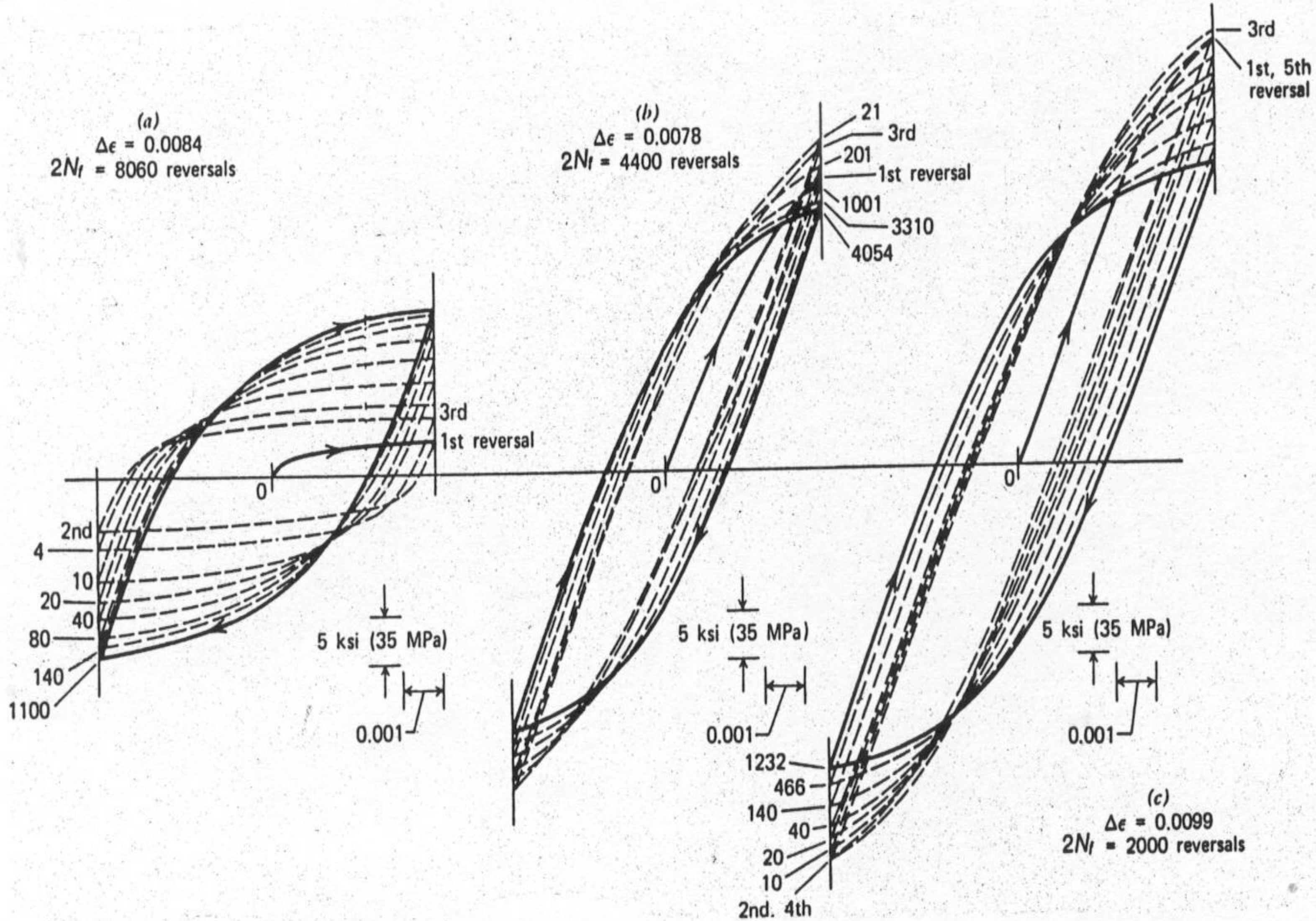
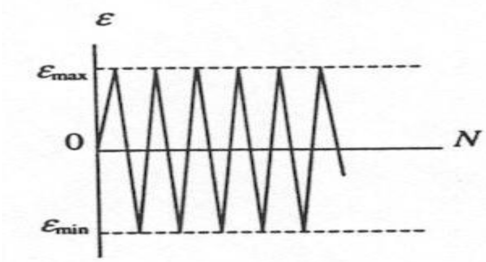
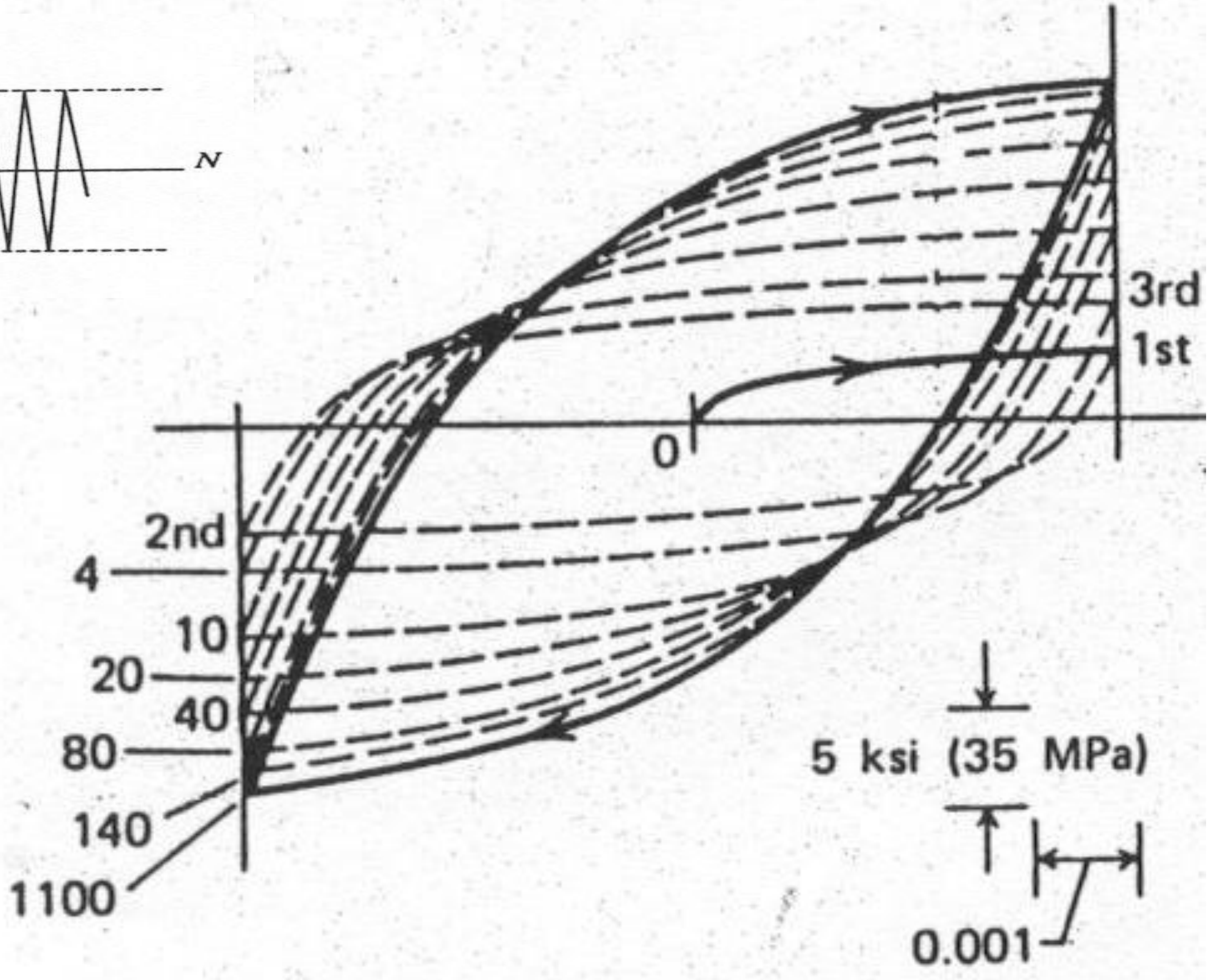


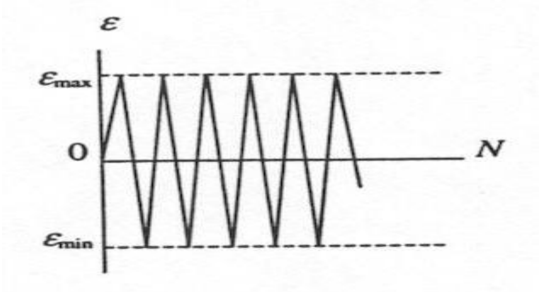
Figure 5.5 Stress–strain behavior of copper subjected to cyclic strain-controlled axial loads. (*a*) Fully annealed, showing cyclic hardening. (*b*) Partially annealed, showing small cyclic hardening and softening. (*c*) Cold-worked, showing cyclic softening [8] (reprinted by permission of the American Society for Testing and Materials).

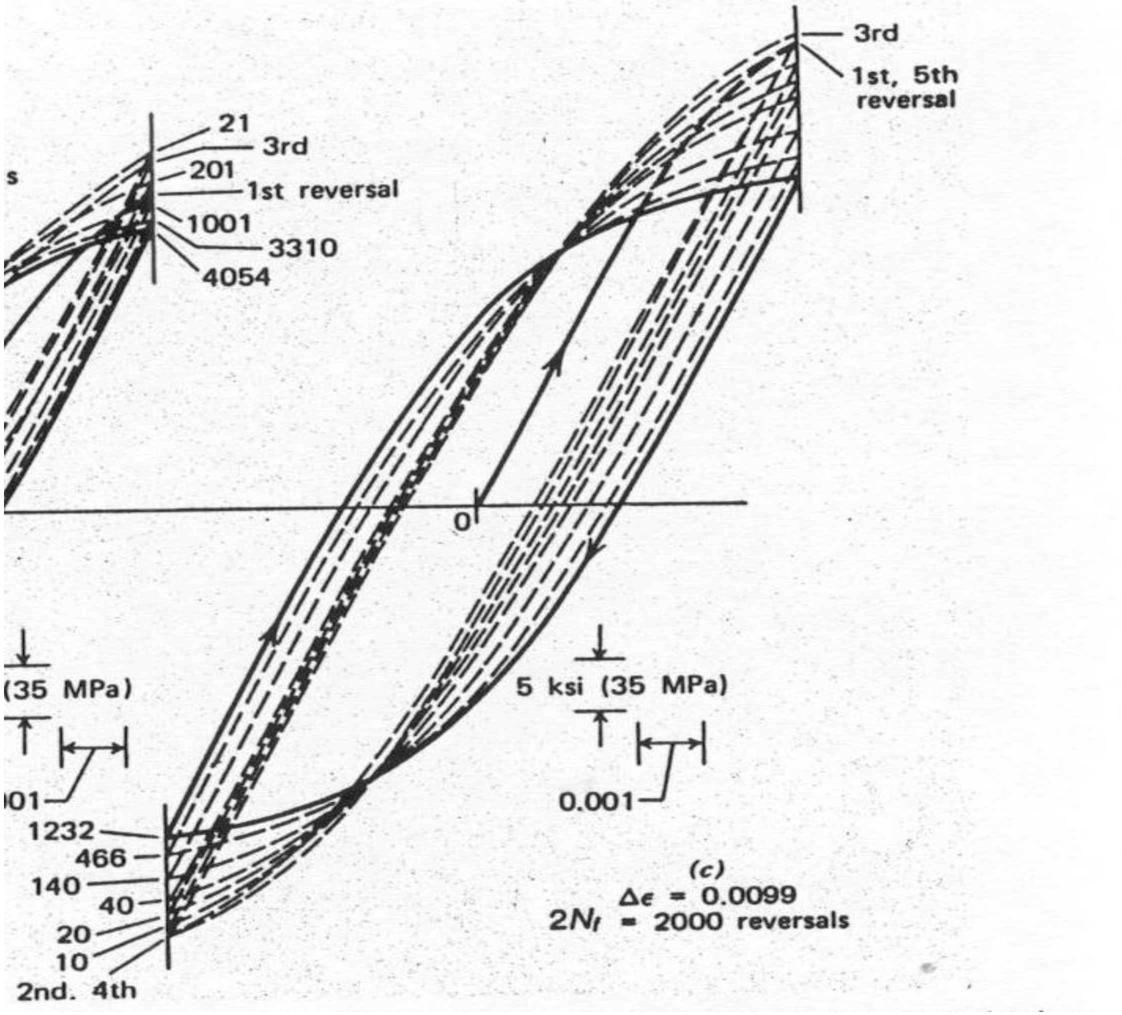
Fully Annealed Copper: **Cyclic Hardening**



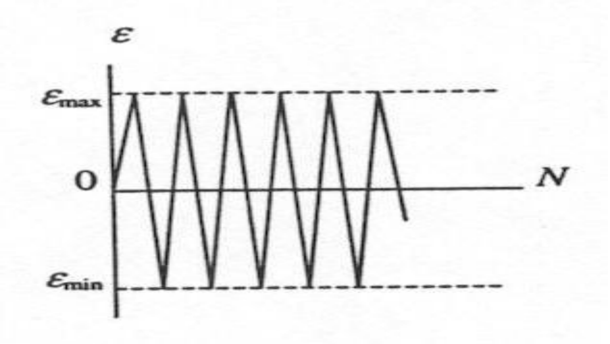


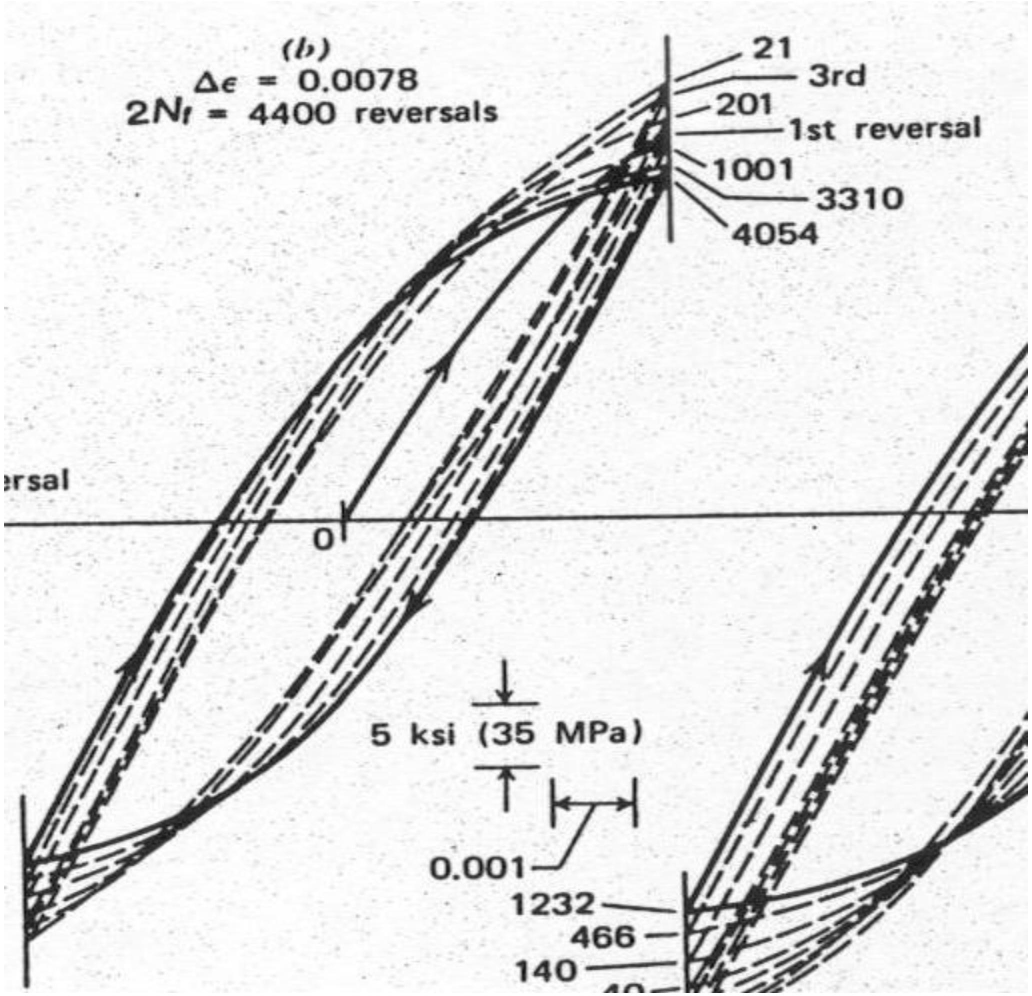
Cold-Worked Copper: **Cyclic Softening**





Partially Annealed Copper:
Cyclic hardening followed by cyclic softening





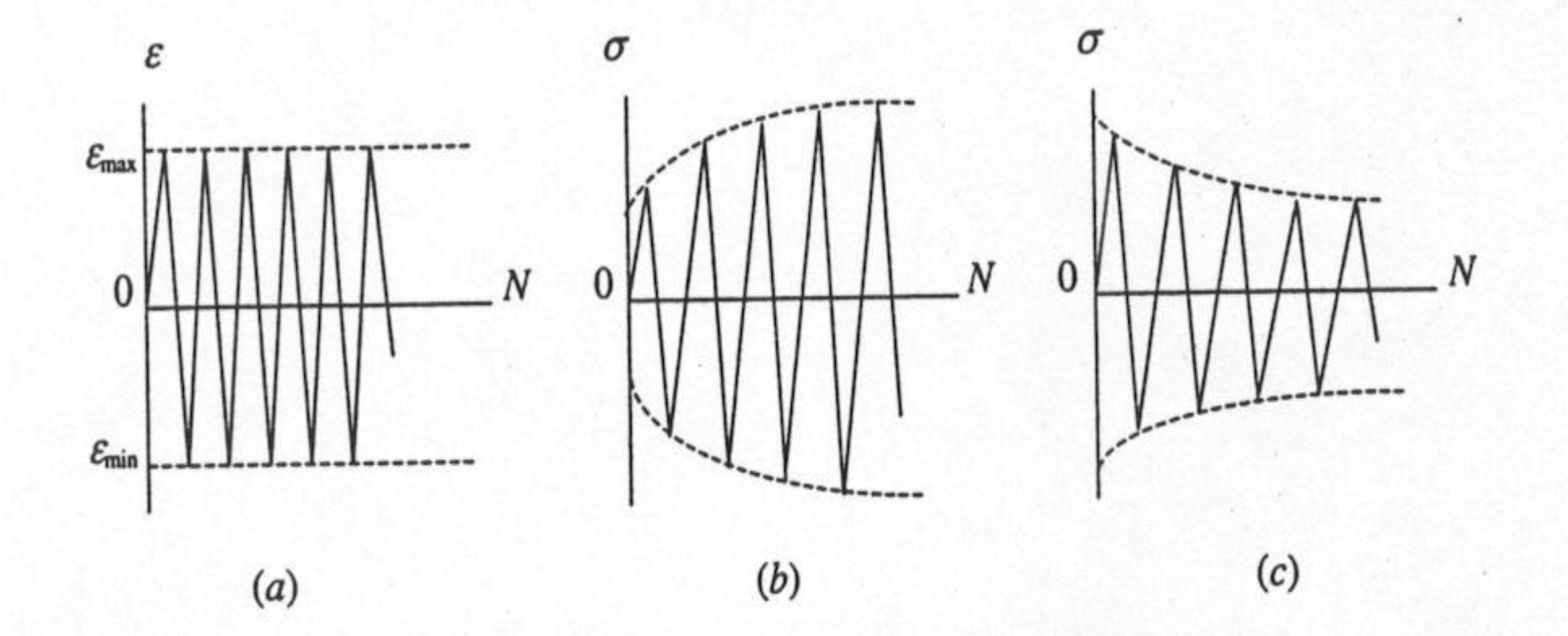
CYCLIC DEFORMATION AND STRESS-STRAIN BEHAVIOR

- The **mechanisms** of hardening and softening were described in terms of dislocation substructure and motion in Section 3.2.

- Changes in cyclic deformation behavior are more pronounced at the beginning of cyclic loading (**transient behavior**), but the material usually gradually stabilizes (**steady-state**) with continued cycling.

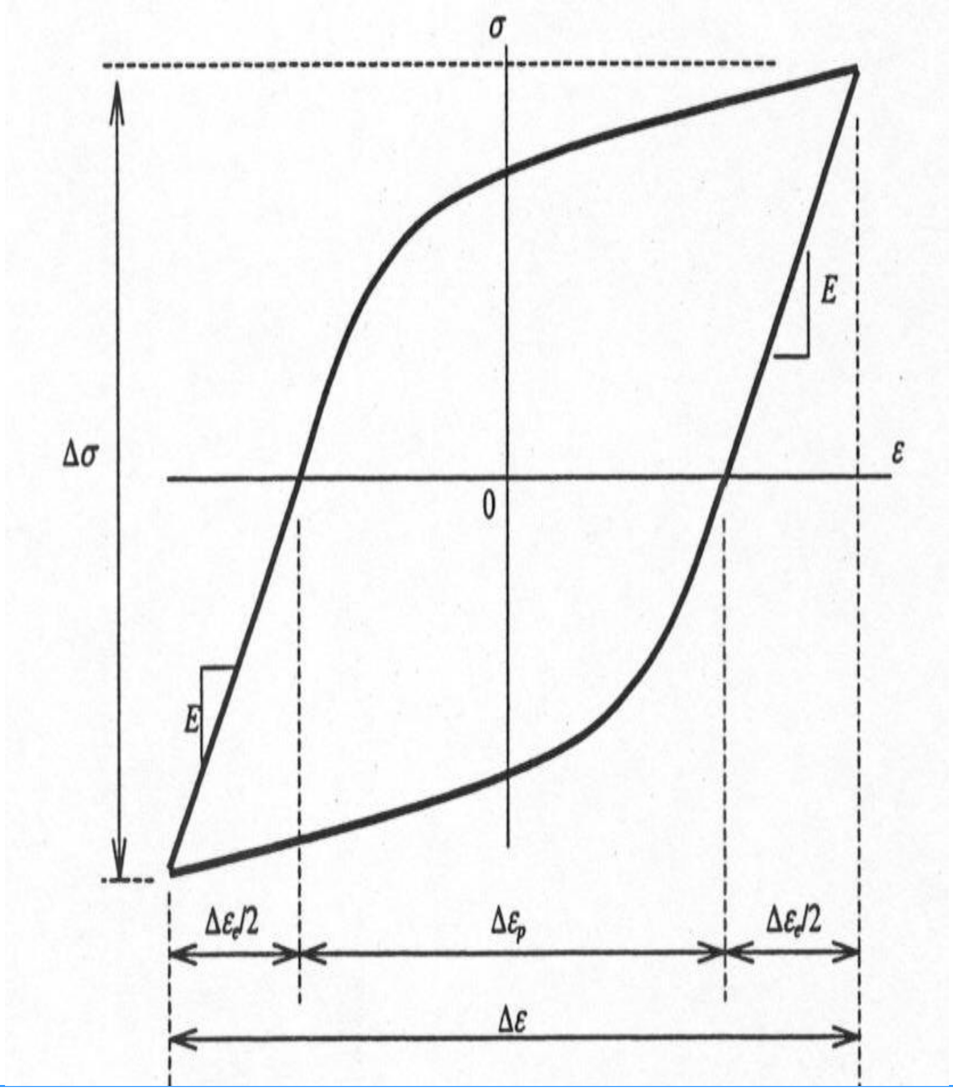
- The **extent and rate of cyclic hardening or softening** under strain-controlled testing conditions can be evaluated by recording stress variation as a function of cycles (Fig. 5.6).

Stress Variation as a Function of Cycles



- Cyclic **hardening** indicates **increased resistance** to deformation.
- Cyclic **softening** indicates **decreased resistance** to deformation.

CYCLIC STRESS-STRAIN BEHAVIOR



- A hysteresis loop from about half the fatigue life is often used to represent the **stable or steady-state** cyclic stress-strain behavior of the material.

$\Delta\varepsilon =$ total true strain range

$\Delta\sigma =$ true stress range

$\Delta\varepsilon_e =$ true elastic strain range $= \Delta\sigma / E$

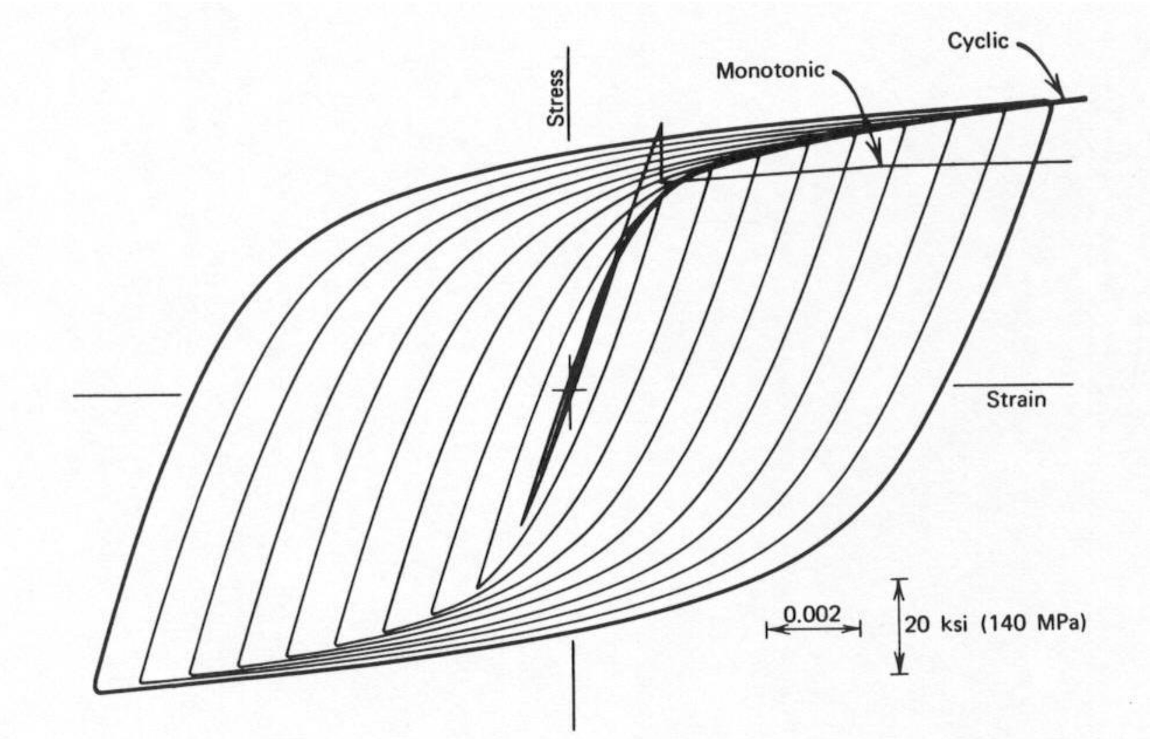
$\Delta\varepsilon_p =$ true plastic strain range

$$\Delta\varepsilon = \Delta\varepsilon_p + \Delta\varepsilon_e = \Delta\varepsilon_p + \frac{\Delta\sigma}{E}$$

CYCLIC STRESS-STRAIN BEHAVIOR

- Even though true stress and strains are used in Figs. 5.6 and 5.7, no distinction is usually made between the true and the engineering values, because:
  - the differences between true and engineering values during the tension and compression parts of the cycle are opposite to each other, and therefore, cancel out,
  - strain levels in cyclic loading applications are often small (typically less than 2%), compared to strain levels in monotonic loading.

CYCLIC STRESS-STRAIN BEHAVIOR



- A family of **stabilized hysteresis loops** at different strain amplitudes is used to obtain the cyclic stress-strain curve of a material.
  - The tips from the family of multiple loops can be connected to form the cyclic stress-strain curve.
  - This curve does not contain the monotonic upper and lower yield points.

CYCLIC STRESS-STRAIN BEHAVIOR

- Three methods commonly used to obtain the cyclic stress-strain curve are:
  - the companion test method
  - the incremental step test method
  - the multiple step test method

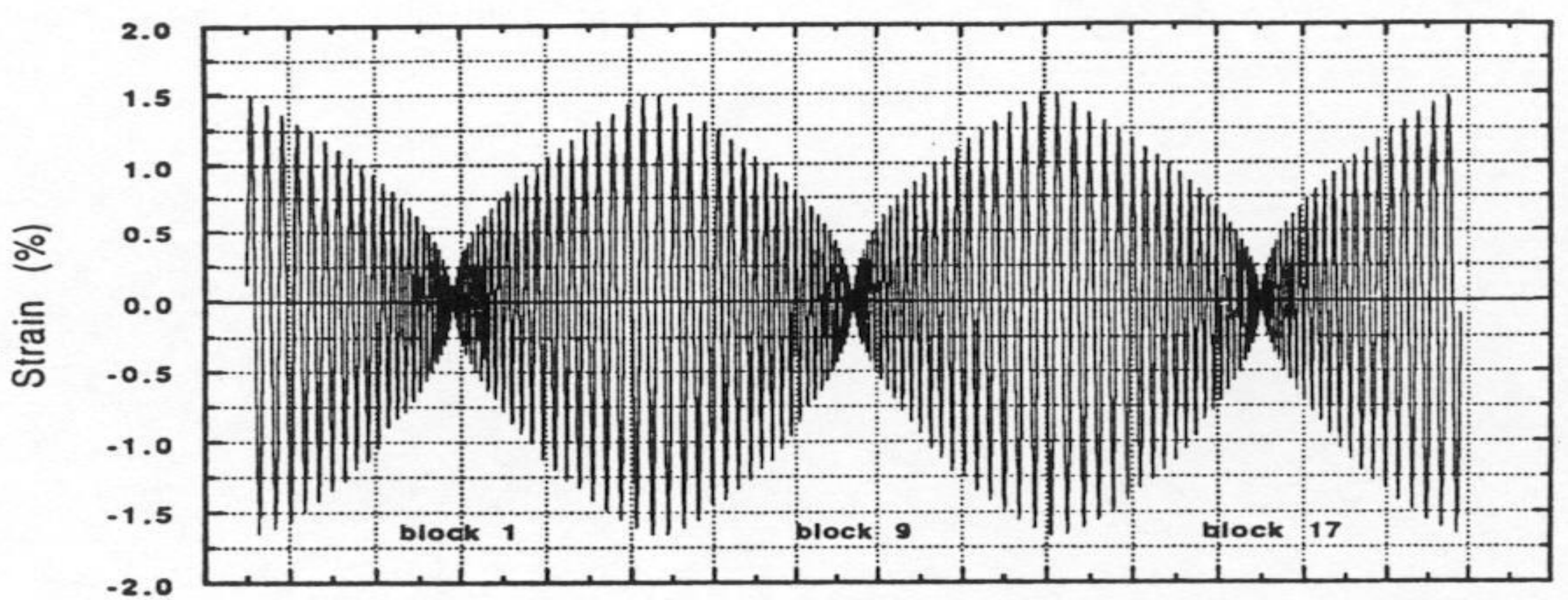
- Even though some differences exist between the results from the three methods, they are small in most cases.

COMPANION TEST METHOD

- Requires a series of test specimens, where each specimen is subjected to a constant strain amplitude until failure.
- Half-life or near half-life hysteresis loops from each specimen and strain amplitude are used to obtain the cyclic stress-strain curve.
- If the experimental program includes strain-controlled fatigue tests, the cyclic stress-strain curve can be obtained from the same fatigue data using the companion method.

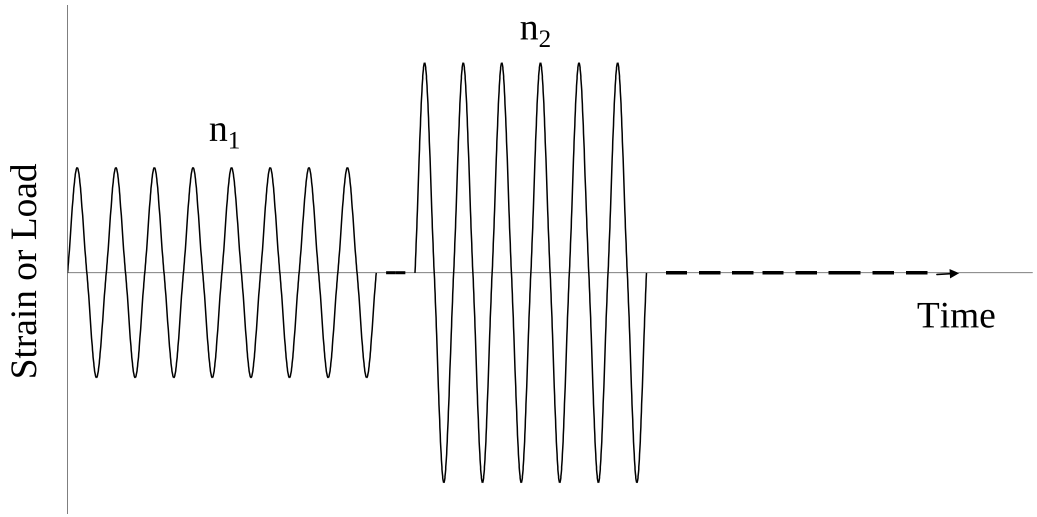
INCREMENTAL STEP METHOD

- A single specimen is subjected to repeated blocks of incrementally increasing and decreasing strains.
- After the material has stabilized (usually after several strain blocks), the hysteresis loops from half a stable block are then used to obtain the cyclic stress-strain curve.



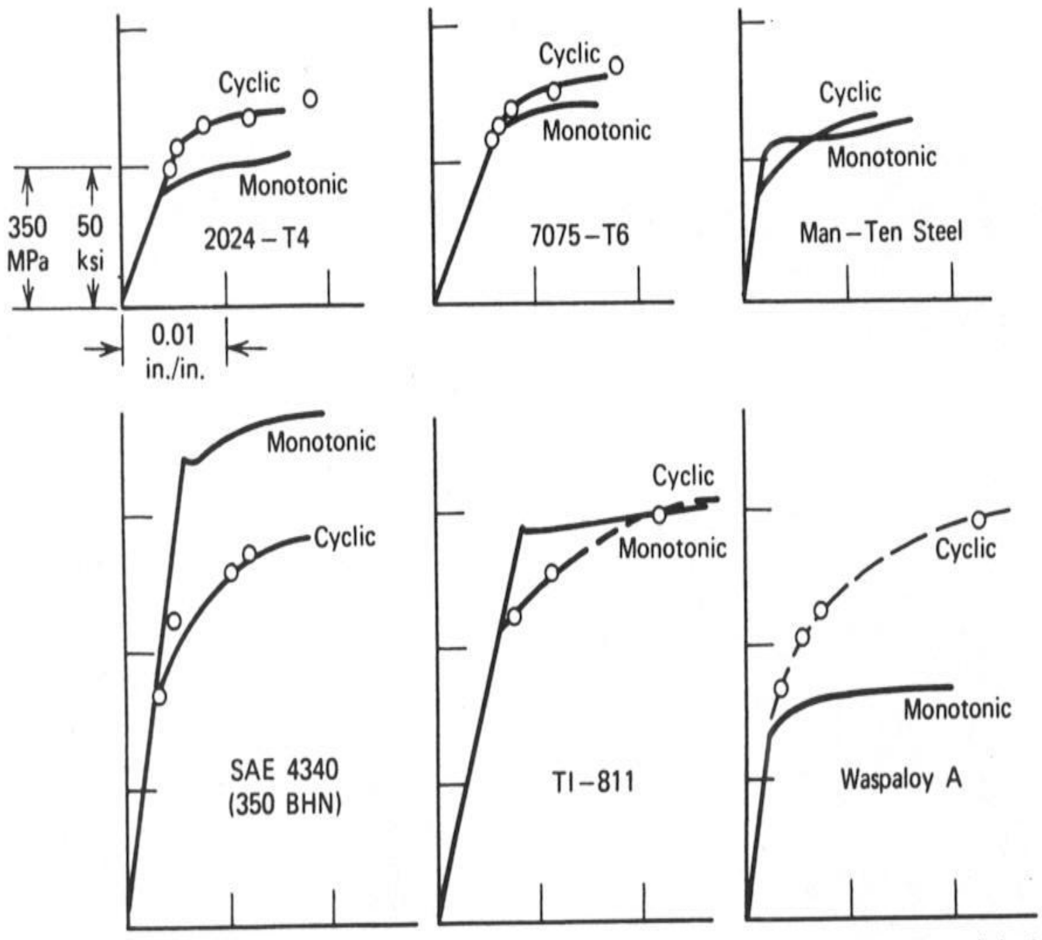
MULTIPLE STEP TEST METHOD

- It is similar to the incremental step test method, except rather than incrementally increasing and decreasing strain in each block the strain amplitude is kept constant.
- Once cyclic stability is reached at the constant strain amplitude, the stable hysteresis loop is recorded and strain amplitude is increased to a higher level.
- This process is repeated until sufficient number of stable hysteresis loops are recorded to construct the cyclic stress-stain curve.



CYCLIC STRESS-STRAIN BEHAVIOR

Cyclic and Monotonic Stress-strain Curves for Several Materials



- **Cyclic softening** exists if the cyclic curve is below the monotonic curve.
  - Using monotonic properties in a cyclic loading application for a cyclic softening material can significantly underestimate the extent of plastic strain which may exist.
- **Cyclic hardening** is present if the cyclic curve lies above the monotonic curve.

CYCLIC STRESS-STRAIN BEHAVIOR

- Similar to the monotonic deformation in a tension test, a plot of true stress amplitude, σ_a, versus true plastic strain amplitude, $\Delta\varepsilon_p/2$, in log-log coordinates for most metals results in a linear curve which is represented by the power function:

$$\sigma_a = K'\left(\frac{\Delta\varepsilon_p}{2}\right)^{n'}$$

K' = cyclic strength coefficient,

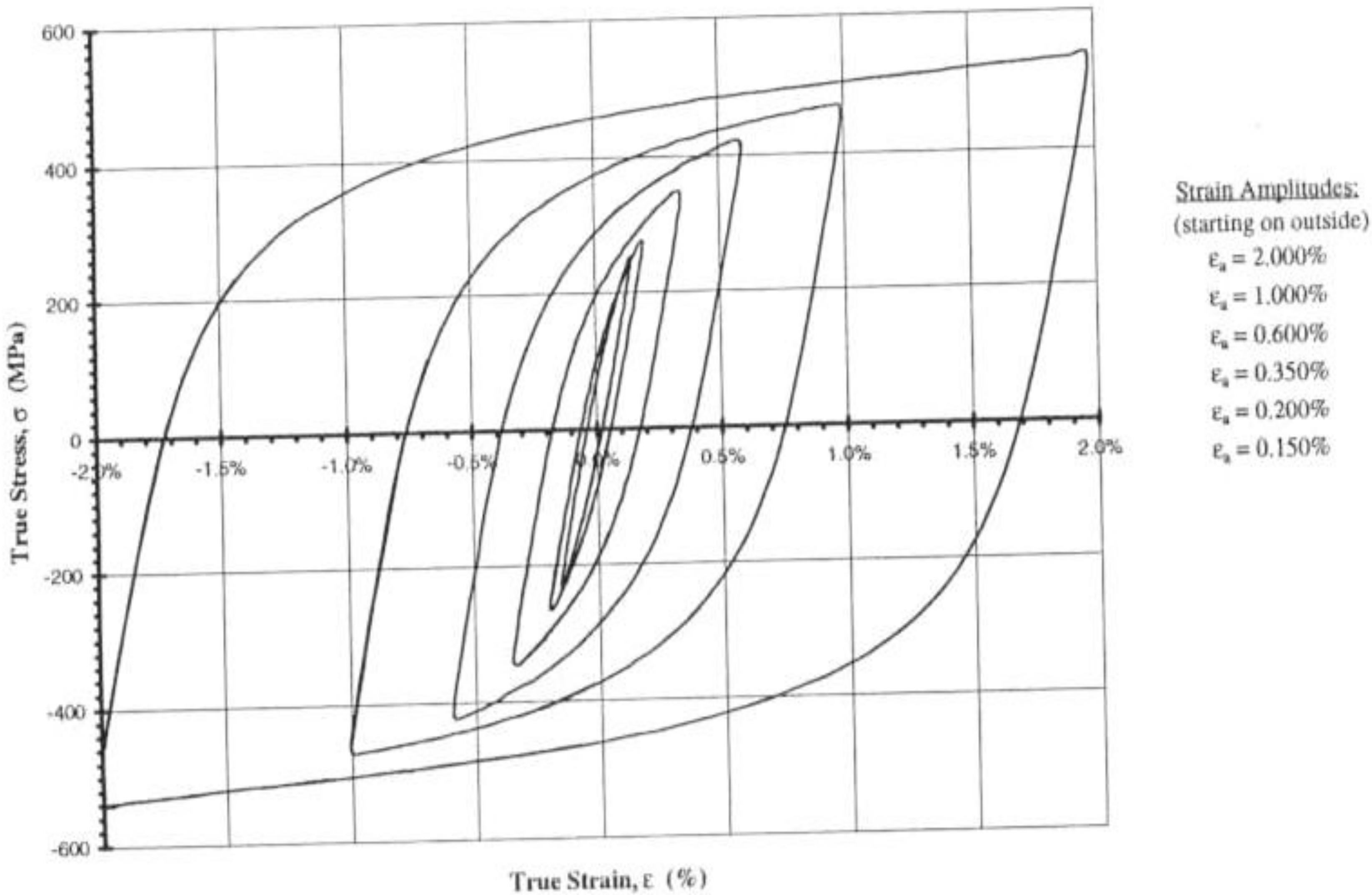
n' = cyclic strain hardening exponent

- The cyclic stress-strain equation represented by a Ramberg-Osgood type relationship is then given by:

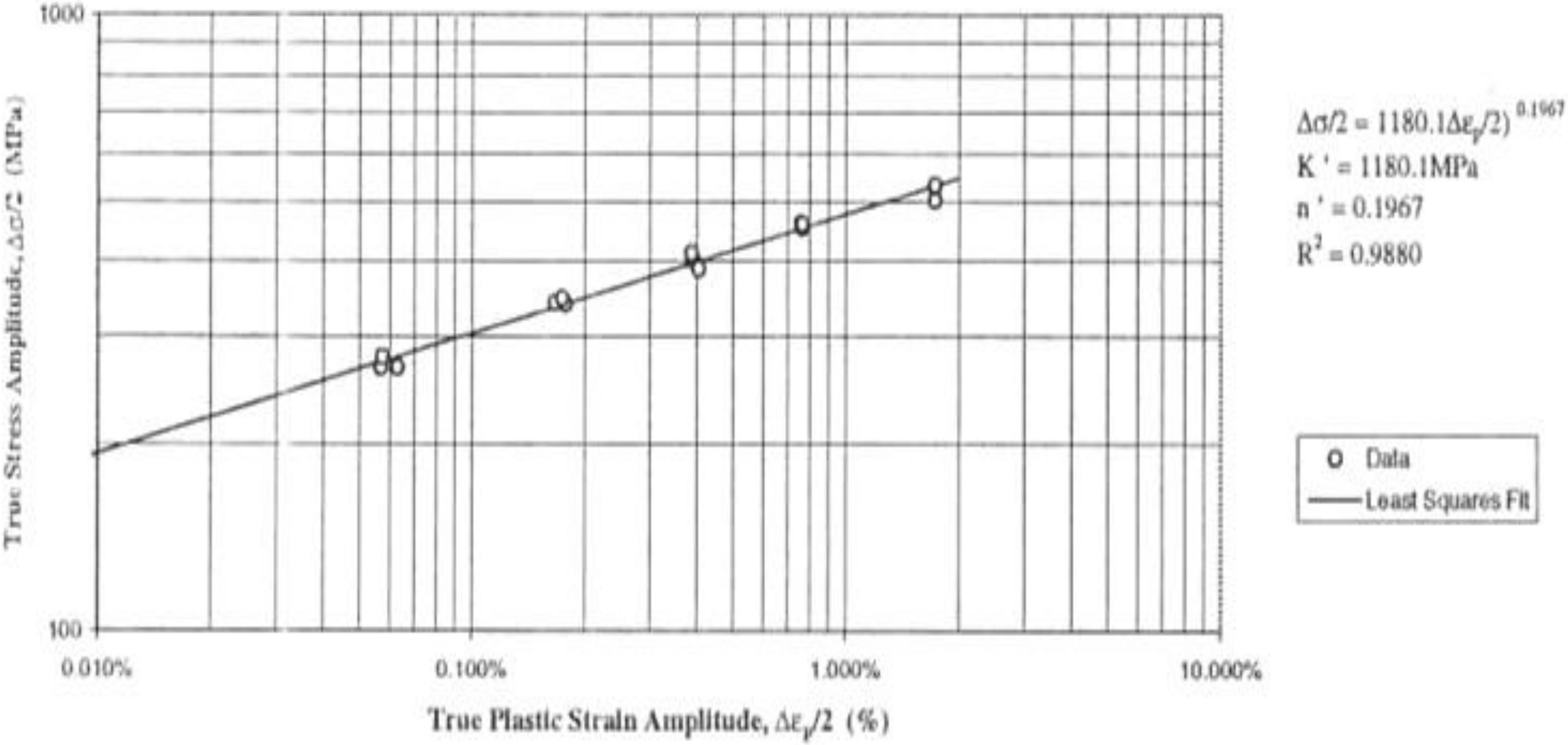
$$\varepsilon_a = \frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{K'}\right)^{1/n'}$$

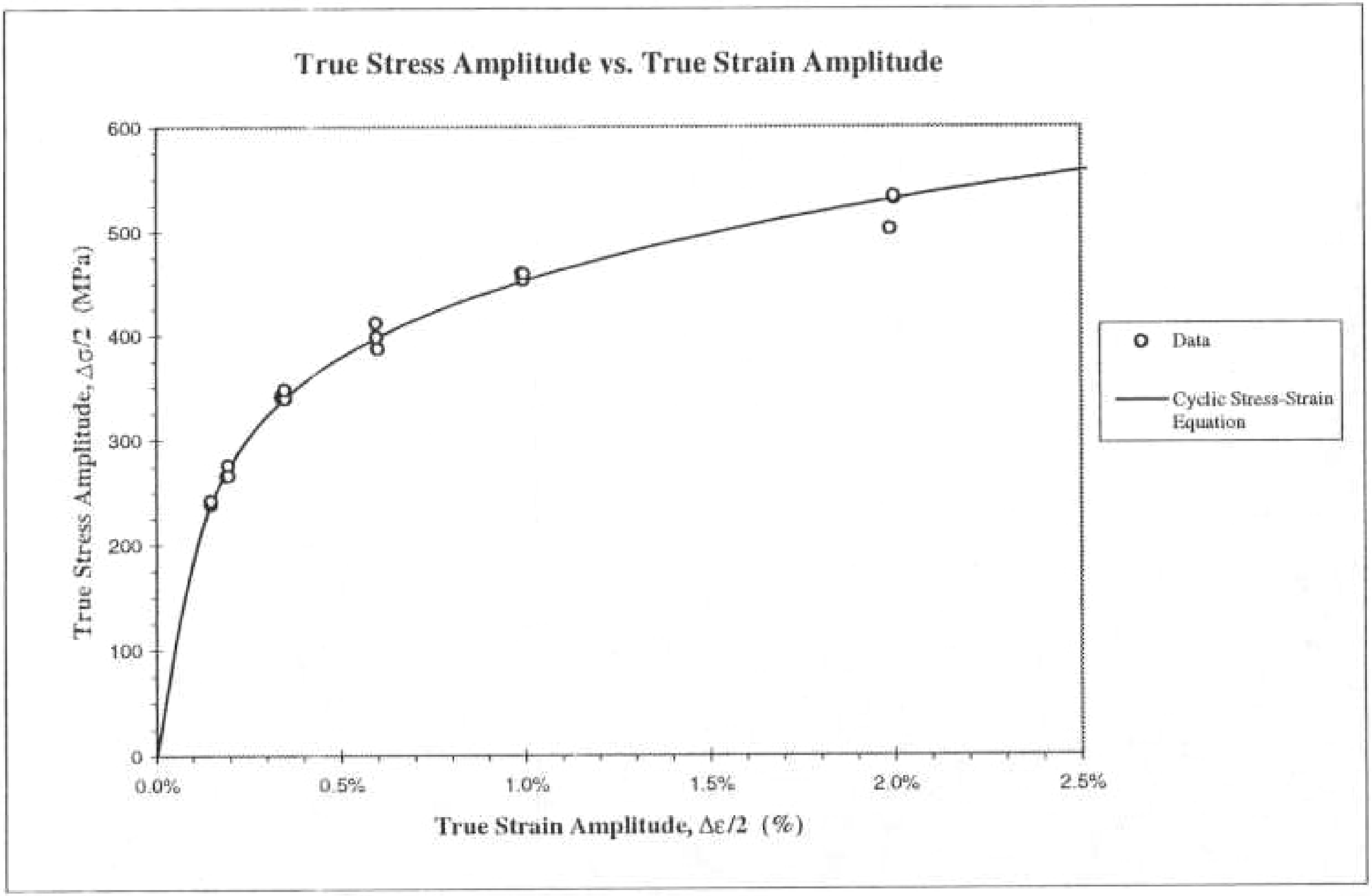
EXAMPLE DATA AND PLOTS

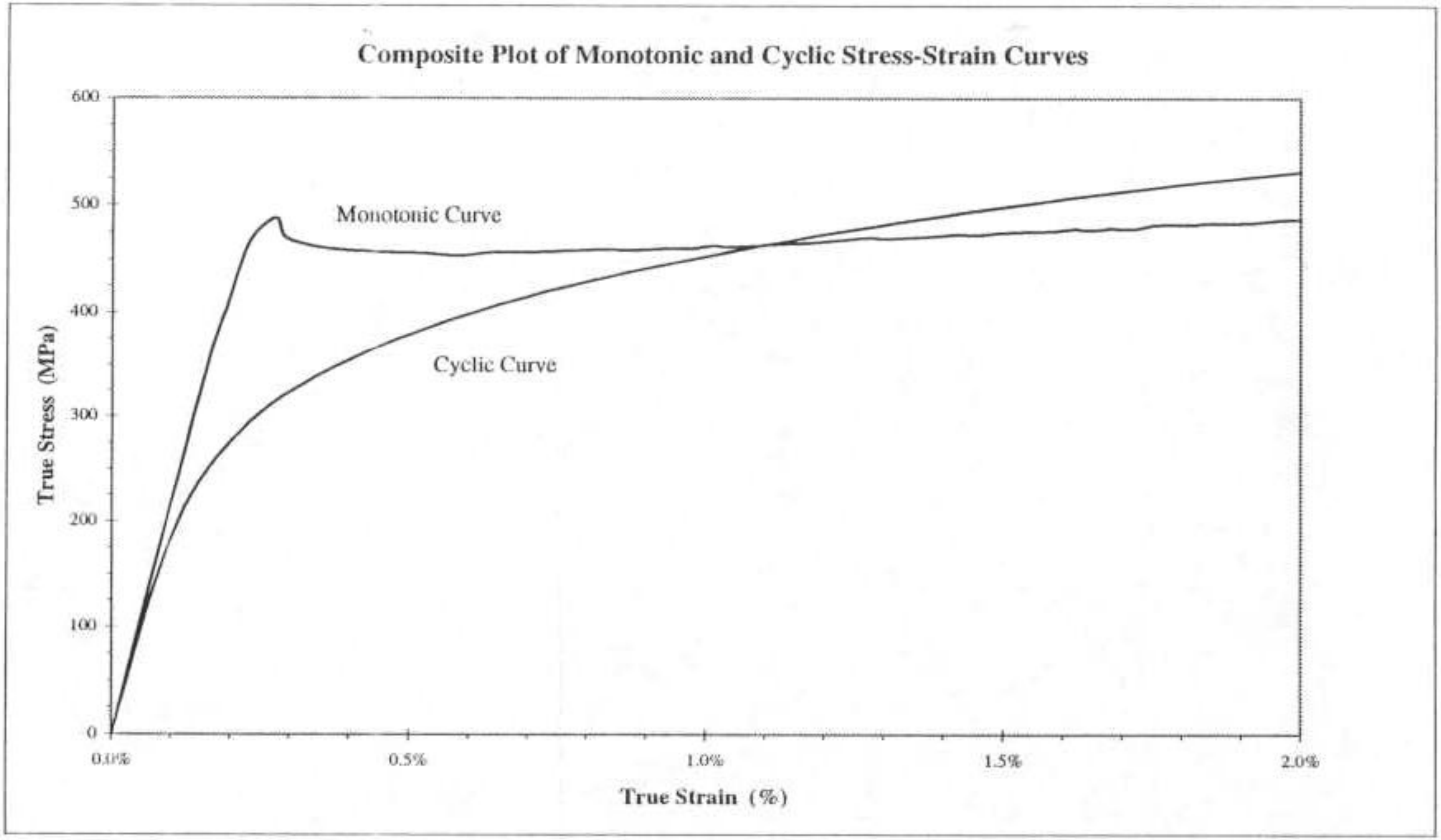
Specimen ID	Test control mode	Test freq., Hz	E, GPa (ksi)	At midlife ($N_{50\%}$) E', GPa (ksi)						2$N_{50\%}$, [a] reversals	$(2N_f)_{50\%}$, [b] reversals	Failure location[c]
				E', GPa (ksi)	$\Delta\varepsilon/2$, %	$\Delta\varepsilon_p/2$ (calculated), %	$\Delta\varepsilon_p/2$ (measured), %	$\Delta\sigma/2$, MPa (ksi)	σ_m, MPa (ksi)			
D9-27	strain	0.20	212.0 (30,744.1)	226.8 (32,889.1)	1.988%	1.735%	1.696%	503.2 (73.0)	-22.9 (-3.3)	400	872	IGL
D9-14	strain	0.20	205.2 (29,760.3)	212.5 (30,826.2)	1.999%	1.733%	1.700%	531.8 (77.1)	1.7 (0.3)	400	852	IGL
D9-5	strain	0.20	206.8 (29,991.3)	195.7 (28,378.5)	1.997%	1.730%	1.708%	533.6 (77.4)	-4.9 (-0.7)	400	816	IGL
D9-16	strain	0.50	205.2 (29,756.1)	196.7 (28,522.4)	0.991%	0.760%	0.743%	460.9 (66.8)	-6.0 (-0.9)	1,280	4,240	IGL
D9-1	strain	0.50	207.2 (30,051.9)	198.9 (28,842.0)	0.995%	0.767%	0.745%	454.4 (65.9)	-0.5 (-0.1)	1,710	3,740	IGL
D9-18	strain	0.50	212.9 (30,870.9)	202.6 (29,376.7)	0.997%	0.766%	0.757%	459.9 (66.7)	6.5 (0.9)	1,660	3,350	IGL
D9-3	strain	0.83	201.1 (29,166.1)	196.0 (28,424.7)	0.596%	0.396%	0.382%	398.6 (57.8)	0.2 (0.0)	4,000	13,794	IGL
D9-8	strain	0.83	191.4 (27,764.6)	222.7 (32,296.8)	0.594%	0.388%	0.388%	411.9 (59.7)	3.8 (0.5)	5,856	12,326	IGL
D9-6	strain	0.83	197.7 (28,673.0)	209.8 (30,434.0)	0.600%	0.406%	0.387%	387.6 (56.2)	1.2 (0.2)	6,026	11,918	IGL
D9-15	strain	2.5	201.0 (29,152.1)	192.6 (27,935.1)	0.339%	0.167%	0.162%	341.7 (49.6)	0.4 (0.1)	40,000	62,444	IGL
D9-13	strain	2.5	211.1 (30,615.1)	207.8 (30,137.7)	0.349%	0.179%	0.177%	340.0 (49.3)	5.5 (0.8)	26,166	52,128	IGL
D9-17	strain	2.5	214.0 (31,030.9)	208.8 (30,276.7)	0.349%	0.175%	0.173%	347.8 (50.4)	2.2 (0.3)	26,350	53,404	IGL
D9-10	strain	3.0	204.5 (29,664.1)	194.3 (28,183.7)	0.191%	0.058%	0.065%	265.6 (38.5)	4.2 (0.6)	200,000	582,794	IGL
D9-4	strain	3.0	203.7 (29,545.5)	198.5 (28,782.8)	0.196%	0.057%	0.071%	276.2 (40.1)	4.1 (0.6)	200,000	391,020	IGL
D9-20	strain	3.0	205.6 (29,819.7)	189.4 (27,471.2)	0.197%	0.063%	0.065%	266.0 (38.6)	6.3 (0.9)	200,000	363,644	IGL
D9-9	strain	3.0	212.5 (30,818.2)	205.9 (29,866.8)	0.149%	0.030%	0.030%	238.0 (34.5)	11.0 (1.6)	470,674	10,000,000	None
	load	30						238.0 (34.5)	0.0 (0.0)			
D9-11	load	30						241.9 (35.1)	0.0 (0.0)		2,836,410	IGL
D9-21	load	30						241.9 (35.1)	0.0 (0.0)		10,000,000	None



600
400
200
0
-200
-400
-600
True Stress, σ (MPa)
-2.0%
-1.5%
-1.0%
-0.5%
0.0%
0.5%
1.0%
1.5%
2.0%
True Strain, ε (%)
Strain Amplitudes:
(starting on outside)
$\varepsilon_a = 2.000\%$
$\varepsilon_a = 1.000\%$
$\varepsilon_a = 0.600\%$
$\varepsilon_a = 0.350\%$
$\varepsilon_a = 0.200\%$
$\varepsilon_a = 0.150\%$


1000
100
0.010%
0.100%
1.000%
10.000%
True Stress Amplitude, Δσ/2 (MPa)
True Plastic Strain Amplitude, Δε_p/2 (%)
Δσ/2 = 1180.1(Δε_p/2) ^0.1967
K' = 1180.1MPa
n' = 0.1967
R² = 0.9880
Data
Least Squares Fit


True Stress Amplitude vs. True Strain Amplitude
600
500
400
300
200
100
0
True Stress Amplitude, Δσ/2 (MPa)
0.0%
0.5%
1.0%
1.5%
2.0%
2.5%
True Strain Amplitude, Δε/2 (%)
Data
Cyclic Stress-Strain Equation


Composite Plot of Monotonic and Cyclic Stress-Strain Curves
Monotonic Curve
Cyclic Curve
True Stress (MPa)
600
500
400
300
200
100
0
0.0%
0.5%
1.0%
1.5%
2.0%
True Strain (%)

CYCLIC STRESS-STRAIN BEHAVIOR

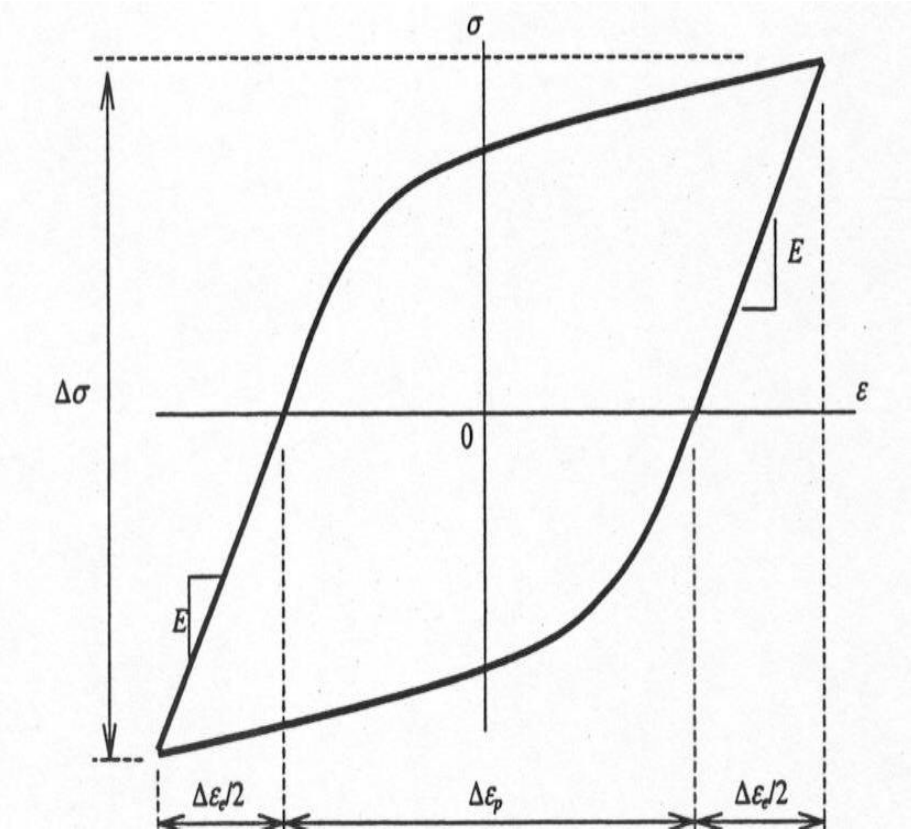
- The **cyclic yield strength, S_y',** is defined at 0.2% strain offset which corresponds to a plastic strain of 0.002 on the cyclic stress-strain curve. It can be estimated by substituting $\Delta\varepsilon_p/2 = 0.002$ in:

$$\sigma_a = K'\left(\frac{\Delta\varepsilon_p}{2}\right)^{n'}$$

- **Values of *K'* and *n'*** for selected engineering alloys are given in Table A.2.

Table A.2 Monotonic, Cyclic, and Strain-life Properties of Selected Engineering Alloys[a,b,c]

Material	Process Description	S_u MPa (ksi)	HB	E GPa (ksi -10^3)	%RA	S_y/S_y' MPa (ksi)	K/K' MPa (ksi)	n/n'	$\varepsilon_f/\varepsilon_f'$	σ_f/σ_f' MPa (ksi)	b	c
Steel												
1010	HR sheet	331	—	203	80	200/—	534/867	0.185/0.244	1.63/0.104	—/499	-0.100	-0.408
		(48)		(29.5)		(29)/ —	(78)/(126)			—/(72)		
1020	HR sheet	441	109	203	62	262/—	738/1962	0.190/0.321	0.96/0.337	—/1384	-0.156	-0.485
		(64)		(29.5)		(38)/ —	(107)/(284)			—/(201)		
1038[c]	Normalized	582	163	201	54	331/342	1106/1340	0.259/0.220	0.77/0.309	898/1043	-0.107	-0.481
		(84)		(29.5)		(48)/(50)	(160)/(195)			(130)/(151)		
1038[c]	Q & T	649	195	219	67	410/364	1183/1330	0.221/0.208	1.10/0.255	1197/1009	-0.097	-0.460
		(94)		(31.5)		(60)/(53)	(172)/(193)			(174)/(146)		
Man-Ten	HR sheet	510	—	207	64	393/372	—/786	0.20/0.11	1.02/0.86	814/807	-0.071	-0.65
		(74)		(30)		(57)/(54)	—/(114)			(118)/(117)		
RQC-100	HR sheet	931	290	207	64	883/600	1172/1434	0.06/0.14	1.02/0.66	1330/1240	-0.07	-0.69
		(135)		(30)		(128)/(87)	(170)/(208)			(193)/(180)		
1045	Annealed	752	225	—	44	517/—	—/1022	—/0.152	0.58/0.486	—/916	-0.079	-0.520
		(109)		—		(75)/—	—/(148)			—/(133)		
1045	Q & T	1827	500	207	51	1689/—	—/3371	0.047/0.145	0.71/0.196	—/2661	-0.093	-0.643
		(265)		(30)		(245)/ —	—/(489)			—/(386)		
1090[c]	Normalized	1090	259	203	14	735/545	1765/1611	0.158/0.174	0.15/0.250	—/1310	-0.091	-0.496
		(158)		(29.5)		(107)/(79)	(256)/(234)			—/(190)		
1090[c]	Q & T	1147	309	217	22	650/627	1895/1873	0.165/0.176	0.24/0.700	—/1878	-0.120	-0.600
		(166)		(31.5)		(94)/(91)	(275)/(272)			—/(273)		
1141[c]	Normalized	789	229	220	47	493/481	1379/1441	0.187/0.177	0.64/0.602	1117/1326	-0.103	-0.581
		(115)		(32)		(72)/(70)	(200)/(209)			(162)/(192)		
1141[c]	Q & T	925	277	227	59	814/591	1205/1277	0.074/0.124	0.88/0.309	1405/1127	-0.066	-0.514
		(134)		(33)		(118)/(86)	(125)/(185)			(204)/(164)		
4142	Q & T	1413	380	207	48	1378/—	—/2266	0.051/0.124	0.65/0.637	—/2143	-0.094	-0.761
		(205)		(30)		(200)/—	—/(387)			—/(311)		
4142	Q & T	1929	475	207	35	1722/—	—/2399	0.048/0.094	0.43/0.331	—/2161	-0.081	-0.854
		(280)		(30)		(250)/—	—/(348)			—/(314)		
4340	HR	827	243	193	43	634/—	—/1337	—/0.168	0.57/0.522	—/1198	-0.095	-0.563
		(120)		(28)		(92)/—	—/(194)			—/(174)		



- For metals with symmetric deformation behavior in tension and compression, the **stabilized hysterisis loop** can be obtained by doubling the size of the cyclic stress-strain curve:

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'}$$

CYCLIC DEFORMATION AND STRESS-STRAIN BEHAVIOR

- The macroscopic material behaviors discussed
  - were obtained from uniaxial test specimens with cross-sectional dimension of about 3 to 10 mm (about 1/8 to 3/8 in.).
  - in most cases gross plastic deformation was involved.

- ***How does all this gross plasticity apply to most service fatigue failures where gross plastic deformation does not exist?***
  - Most fatigue failures begin at local discontinuities where **local plasticity** exists and crack nucleation and growth are governed by local plasticity at the notch tip.
  - The type of behavior shown/discussed for gross plastic deformation is similar to that which occurs **locally at notches and crack tips**, and thus is extremely important in fatigue of components and structures.

STRAIN-BASED APPROACH TO LIFE ESTIMATION, ε-*N*

STRAIN-LIFE FATIGUE APPROACH

DETERMINATION OF STRAIN-LIFE FATIGUE PROPERTIES

MEAN STRESS EFFECTS

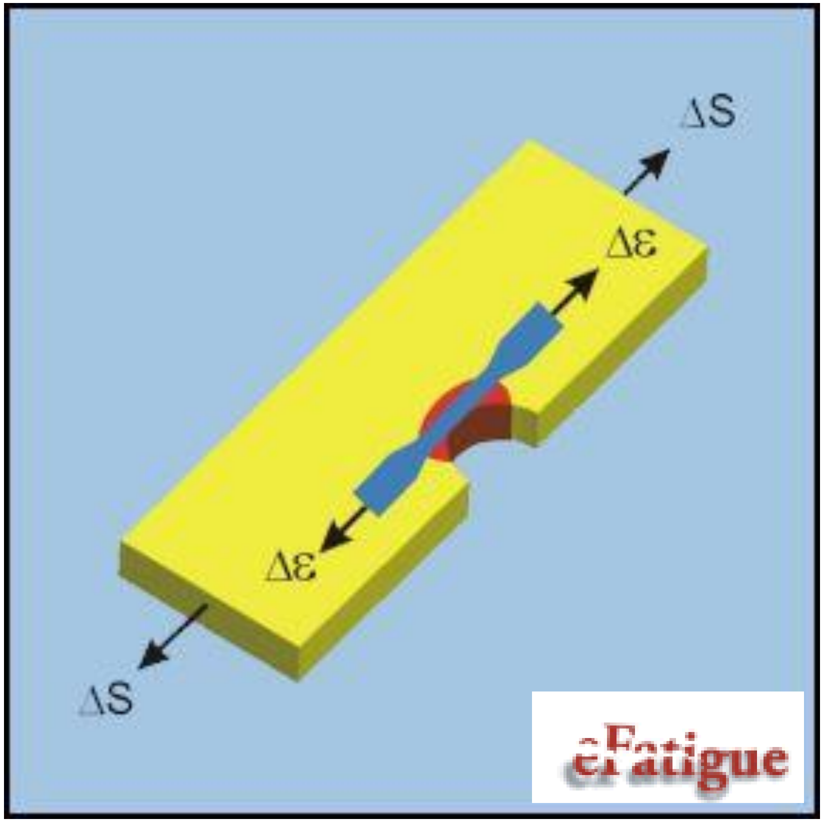
FACTORS INFLUENCING STRAIN-LIFE BEHAVIOR

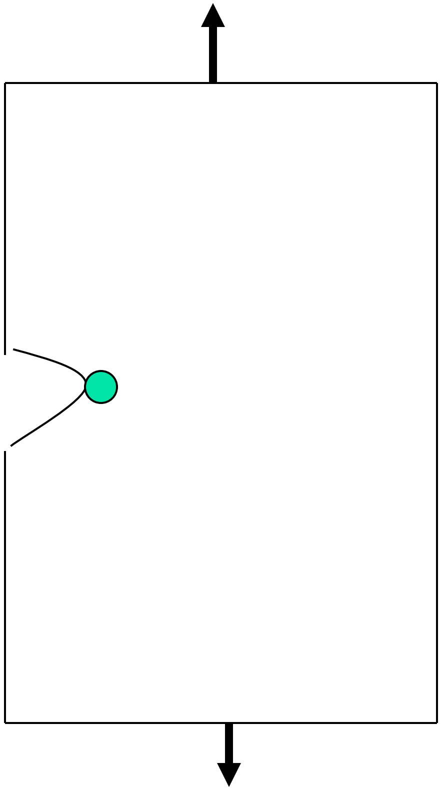
ε -*N* **APPROACH** TO LIFE ESTIMATION

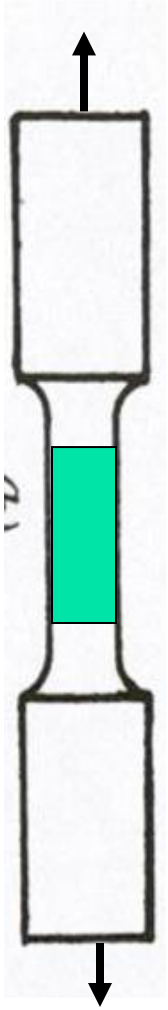
- The strain-based approach to fatigue problems:
  - Is widely used at present.
  - Strain can be directly measured.
  - Application of this approach is common in notched member fatigue.

- Strain-life design method is based on relating the fatigue life of notched parts to the life of small unnotched specimens cycled to the same strains as the material at the notch root.
  - Since fatigue damage is assessed directly in terms of local strain, this approach is also called local strain approach.
  - Expected fatigue life can be determined knowing the strain-time history at the notch root and smooth strain-life fatigue properties of the material.
  - The remaining fatigue crack growth life of a component can be analyzed using fracture mechanics concepts (Ch 6).

LOCAL STRAIN APPROACH







ε -*N* APPROACH TO LIFE ESTIMATION

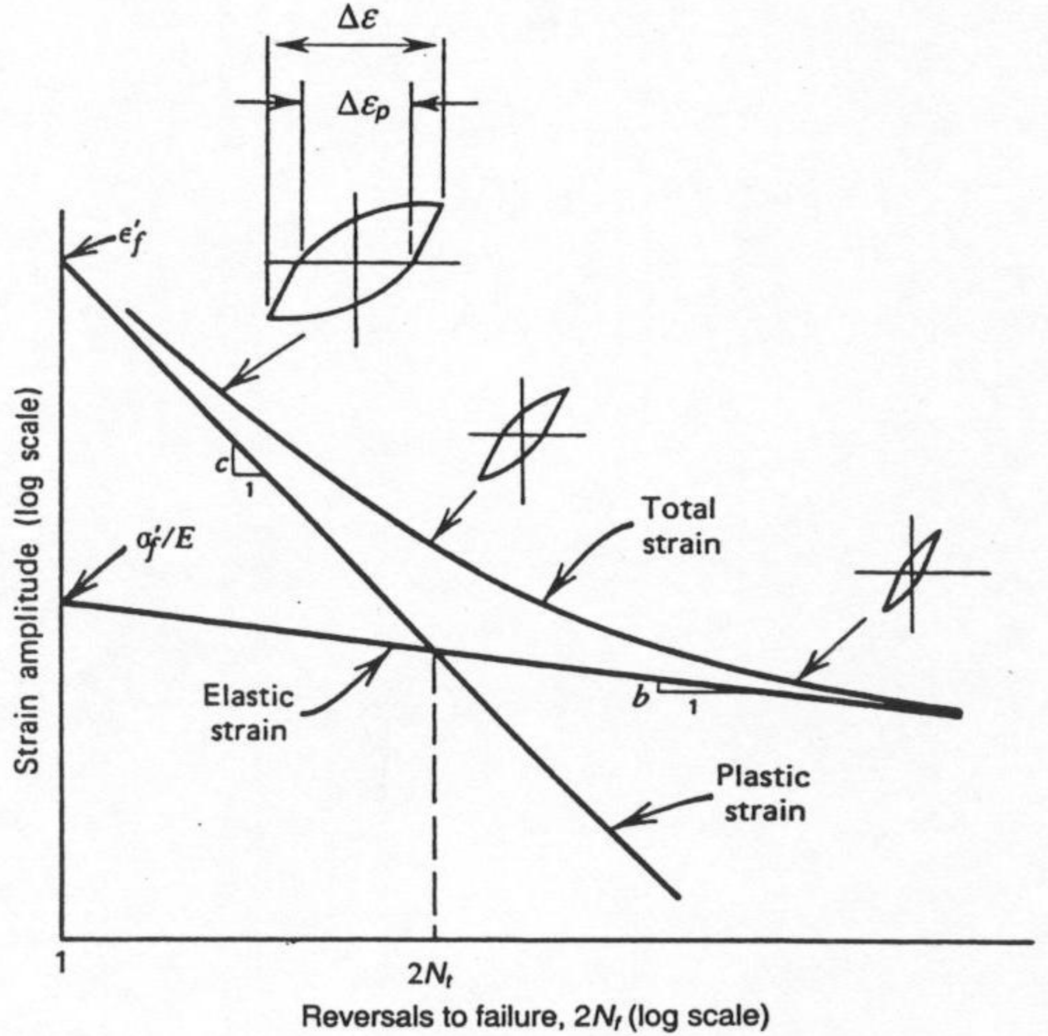
- Steady-state hysteresis loops can be reduced to elastic and plastic strain ranges or amplitudes.
- Cycles to failure can involve from about 10 to 10^7 cycles and frequencies can range from about 0.1 to 10 Hz.
- Beyond 10^6 cycles, load or stress controlled tests at higher frequencies can often be performed because of the small plastic strains and the greater time to failure.
- The strain-life curves are often called **low cycle fatigue** data because much of the data are for less than 10^5 cycles.

ε -*N* APPROACH TO LIFE ESTIMATION

- Failure criteria include
  - the life to a small detectable crack,
  - life to a certain percentage decrease in load amplitude (50% drop level is recommended by the ASTM standard E606),
  - life to a certain decrease in the ratio of unloading to loading moduli, or
  - life to fracture.

- Strain-life fatigue curves plotted on log-log scales are shown schematically in Fig. 5.11.

- The total strain amplitude can be resolved into elastic and plastic strain components from the steady-state hysteresis loops.
- Both the elastic and plastic curves can be approximated as straight lines.
- At large strains or short lives, the plastic strain component is predominant, and at small strains or longer lives the elastic strain component is predominant.



ε -*N* APPROACH TO LIFE ESTIMATION

- The straight line elastic behavior can be transformed to Basquin's eqn:

$$\frac{\Delta\sigma}{2} = \sigma_a = \sigma'_f \left(2N_f\right)^b$$

- The relation between plastic strain and life is (Manson-Coffin relationship):

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon'_f \left(2N_f\right)^c$$

- The intercepts of the two straight lines at $2N_f = 1$ are σ'_f/E for the elastic component and ε'_f for the plastic component.

- The slopes of the elastic and plastic lines are ***b*** and ***c***, respectively.

ε -N APPROACH TO LIFE ESTIMATION

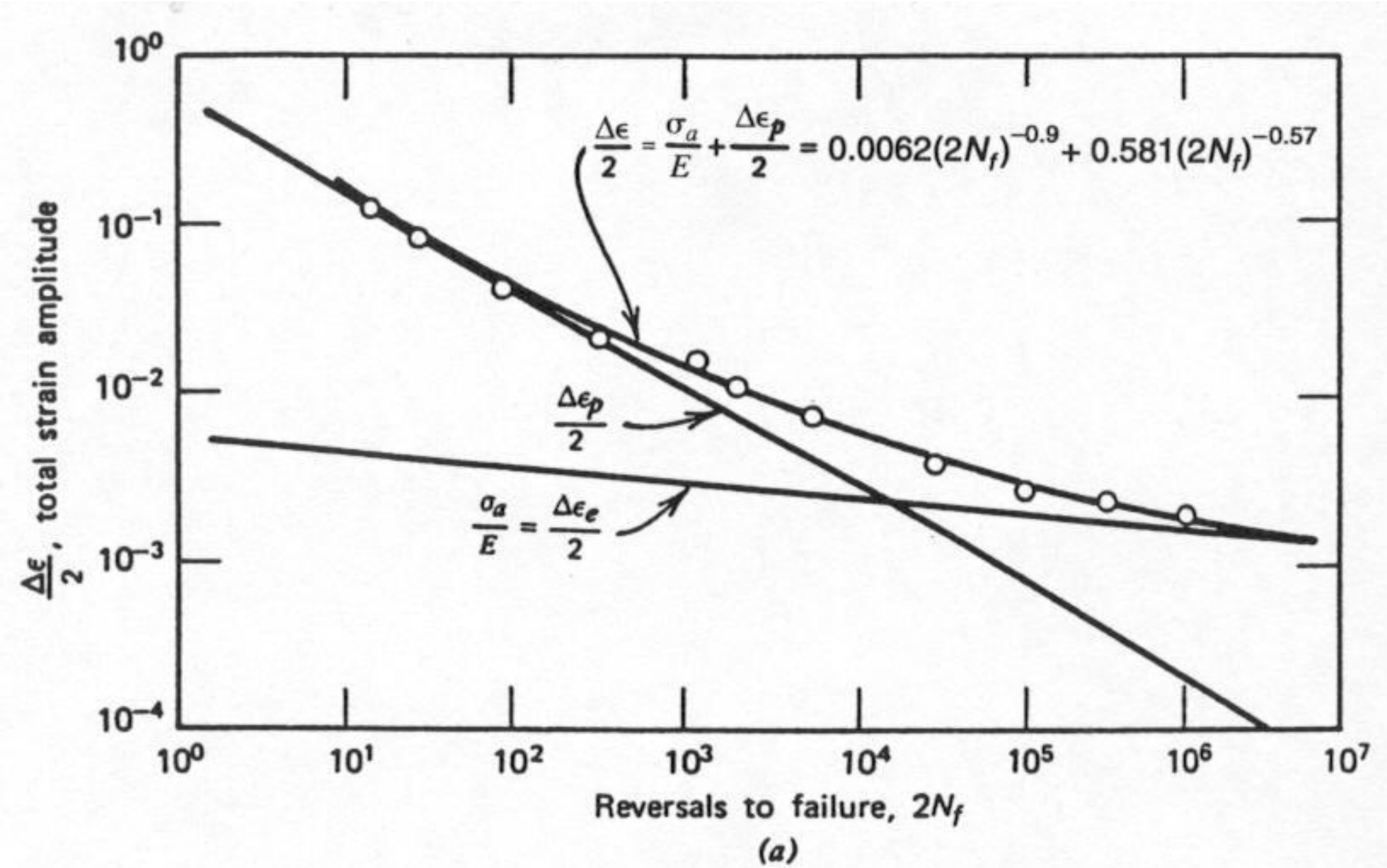
Therefore:

$$\frac{\Delta\varepsilon}{2} = \varepsilon_a = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = \frac{\sigma'_f}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c$$

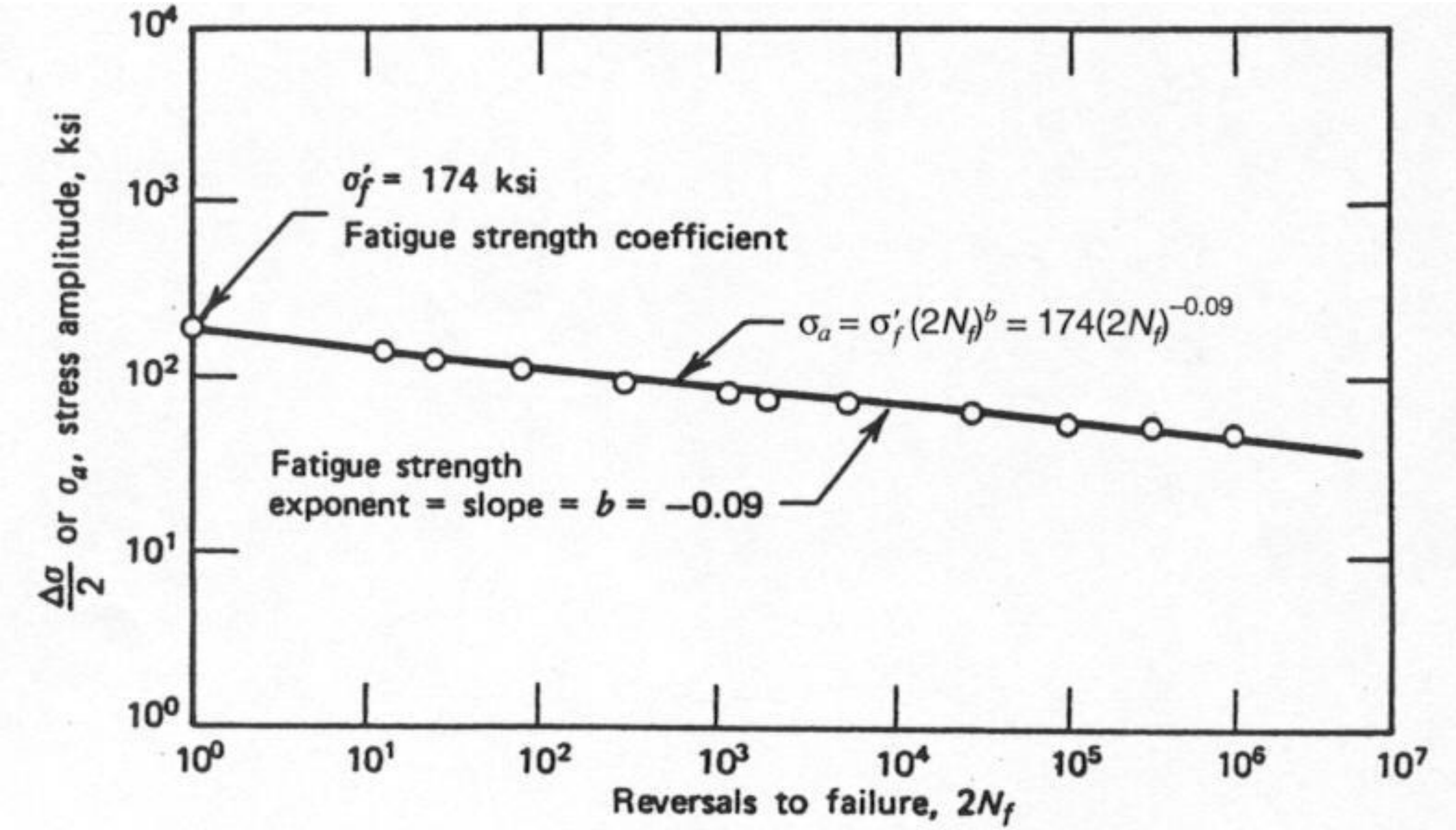
Where:

$\Delta\varepsilon/2$	=	total strain amplitude = ε_a
$\Delta\varepsilon_e/2$	=	elastic strain amplitude = $\Delta\sigma/2E = \sigma_a/E$
$\Delta\varepsilon_p/2$	=	plastic strain amplitude = $\Delta\varepsilon/2 - \Delta\varepsilon_e/2$
ε'_f	=	fatigue ductility coefficient
c	=	fatigue ductility exponent
σ'_f	=	fatigue strength coefficient
b	=	fatigue strength exponent
E	=	modulus of elasticity
$\Delta\sigma/2$	=	σ_a = stress amplitude

A Typical Complete Strain-life Curve With Data Points For 4340 Steel

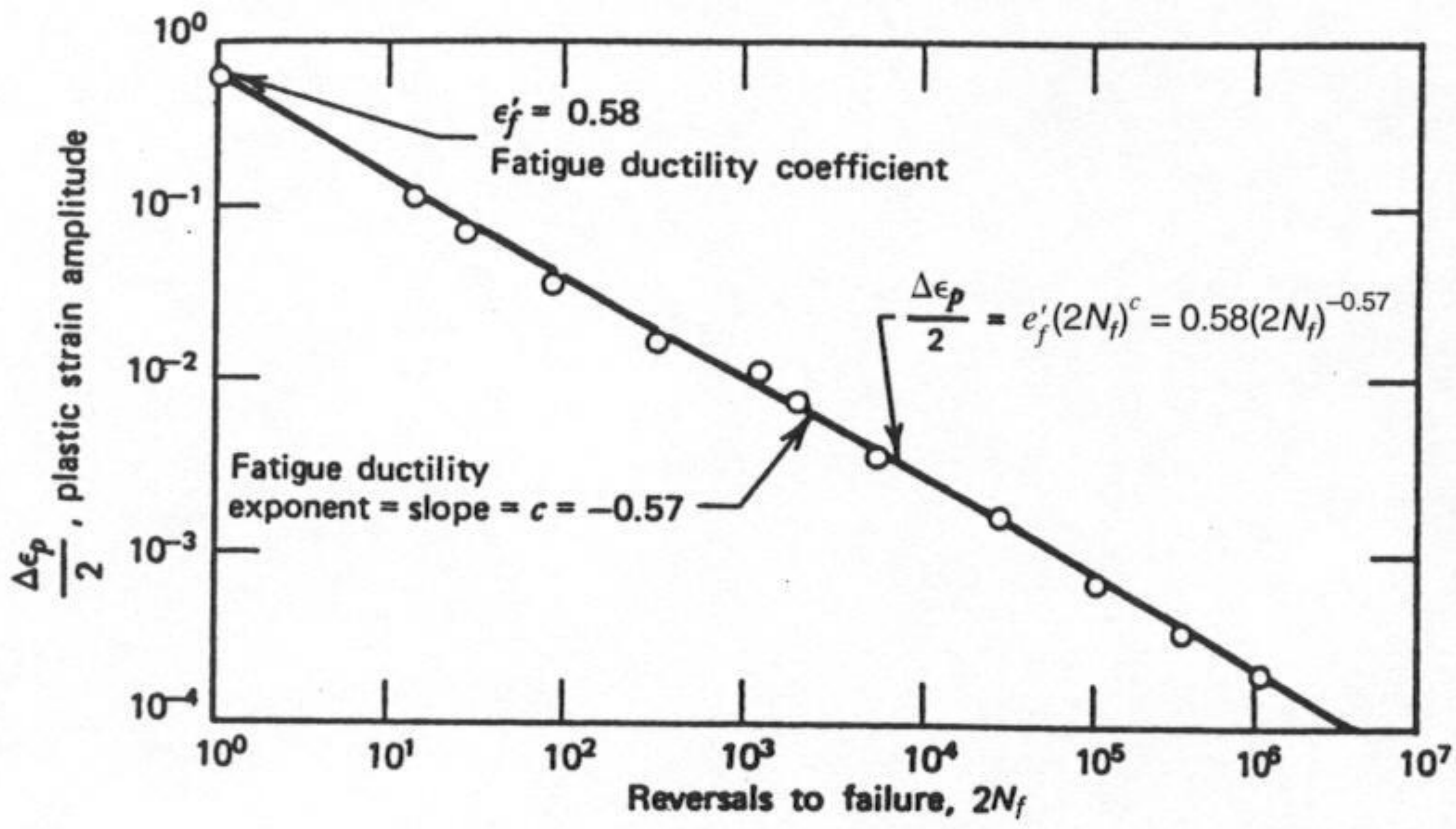


ELASTIC LINE



(b)

PLASTIC LINE

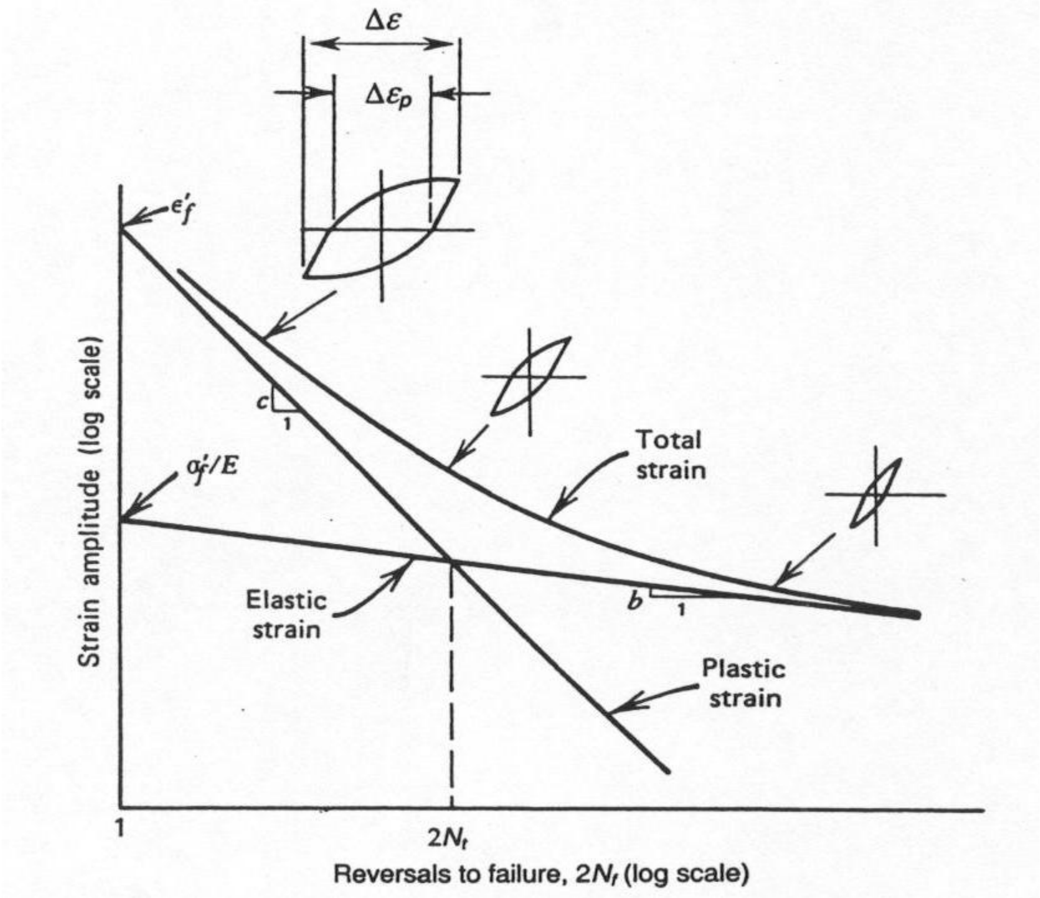


(c)

The life where elastic and plastic components of strain are equal is called the **transition fatigue life**:

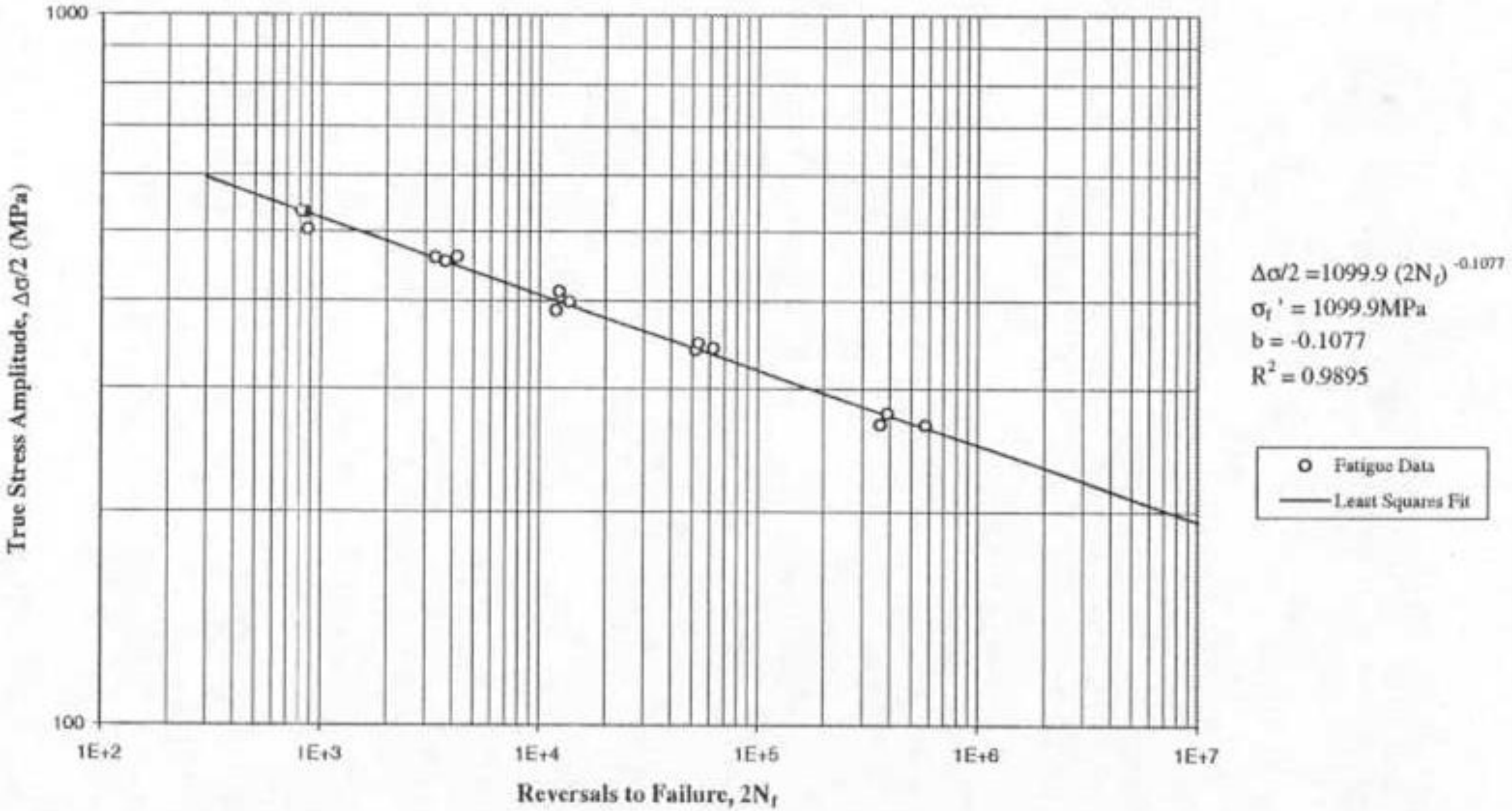
$$2N_t = \left(\frac{\varepsilon'_f E}{\sigma'_f} \right)^{\frac{1}{b-c}}$$

For lives less than $2N_t$ the deformation is mainly plastic, whereas for lives larger than $2N_t$ the deformation is mainly elastic.

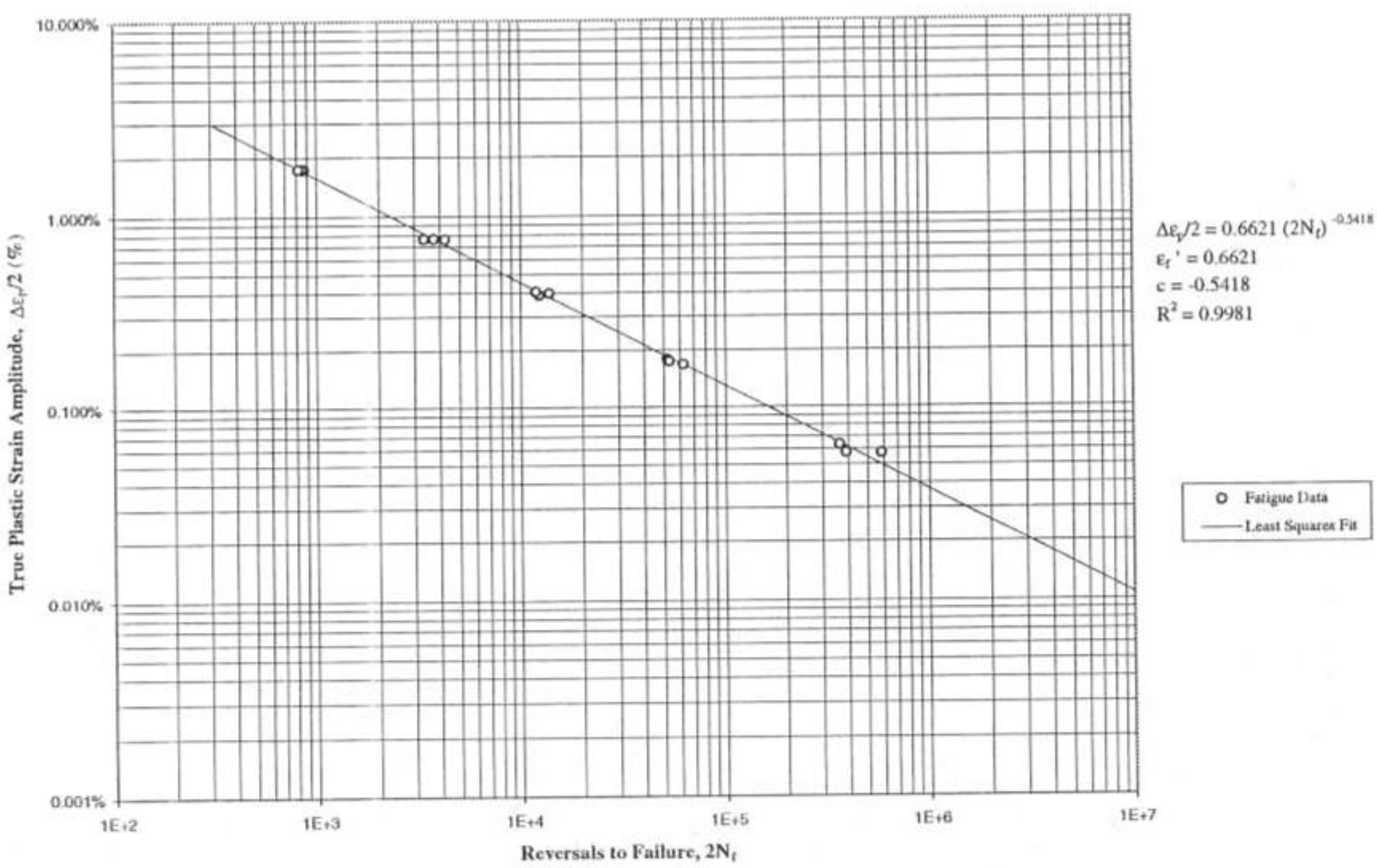


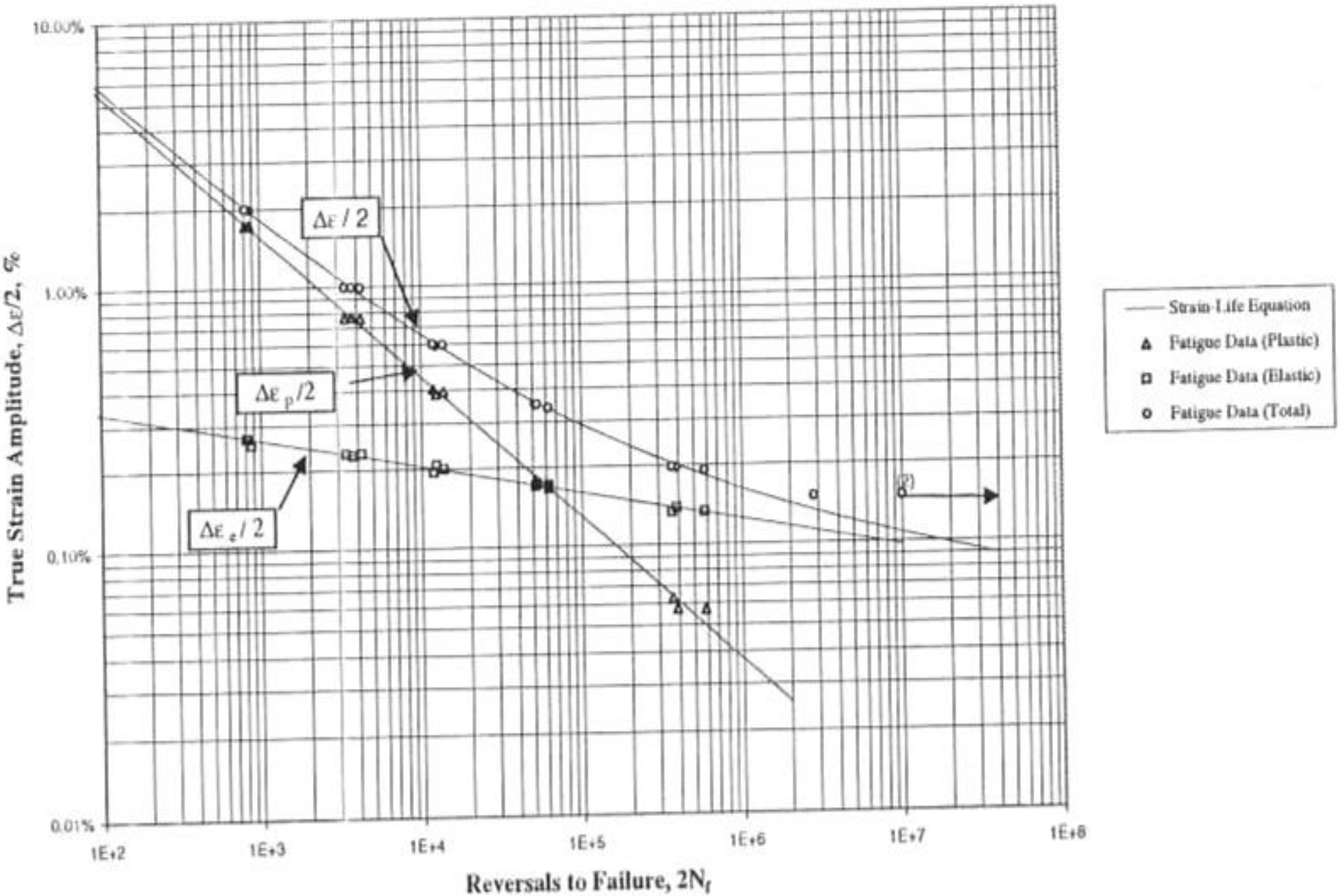
EXAMPLE DATA AND PLOTS

Specimen ID	Test control mode	Test freq., Hz	E, GPa (ksi)	At midlife ($N_{50\%}$) E', GPa (ksi)	$\Delta\varepsilon/2$, %	$\Delta\varepsilon_p/2$ (calculated), %	$\Delta\varepsilon_p/2$ (measured), %	$\Delta\sigma/2$, MPa (ksi)	σ_m, MPa (ksi)	$2N_{50\%}$, [a] reversals	$(2N_f)_{50\%}$, [b] reversals	Failure location [c]
D9-27	strain	0.20	212.0 (30,744.1)	226.8 (32,889.1)	1.988%	1.735%	1.696%	503.2 (73.0)	-22.9 (-3.3)	400	872	IGL
D9-14	strain	0.20	205.2 (29,760.3)	212.5 (30,826.2)	1.999%	1.733%	1.700%	531.8 (77.1)	1.7 (0.3)	400	852	IGL
D9-5	strain	0.20	206.8 (29,991.3)	195.7 (28,378.5)	1.997%	1.730%	1.708%	533.6 (77.4)	-4.9 (-0.7)	400	816	IGL
D9-16	strain	0.50	205.2 (29,756.1)	196.7 (28,522.4)	0.991%	0.760%	0.743%	460.9 (66.8)	-6.0 (-0.9)	1,280	4,240	IGL
D9-1	strain	0.50	207.2 (30,051.9)	198.9 (28,842.0)	0.995%	0.767%	0.745%	454.4 (65.9)	-0.5 (-0.1)	1,710	3,740	IGL
D9-18	strain	0.50	212.9 (30,870.9)	202.6 (29,376.7)	0.997%	0.766%	0.757%	459.9 (66.7)	6.5 (0.9)	1,660	3,350	IGL
D9-3	strain	0.83	201.1 (29,166.1)	196.0 (28,424.7)	0.596%	0.396%	0.382%	398.6 (57.8)	0.2 (0.0)	4,000	13,794	IGL
D9-8	strain	0.83	191.4 (27,764.6)	222.7 (32,296.8)	0.594%	0.388%	0.388%	411.9 (59.7)	3.8 (0.5)	5,856	12,326	IGL
D9-6	strain	0.83	197.7 (28,673.0)	209.8 (30,434.0)	0.600%	0.406%	0.387%	387.6 (56.2)	1.2 (0.2)	6,026	11,918	IGL
D9-15	strain	2.5	201.0 (29,152.1)	192.6 (27,935.1)	0.339%	0.167%	0.162%	341.7 (49.6)	0.4 (0.1)	40,000	62,444	IGL
D9-13	strain	2.5	211.1 (30,615.1)	207.8 (30,137.7)	0.349%	0.179%	0.177%	340.0 (49.3)	5.5 (0.8)	26,166	52,128	IGL
D9-17	strain	2.5	214.0 (31,030.9)	208.8 (30,276.7)	0.349%	0.175%	0.173%	347.8 (50.4)	2.2 (0.3)	26,350	53,404	IGL
D9-10	strain	3.0	204.5 (29,664.1)	194.3 (28,183.7)	0.191%	0.058%	0.065%	265.6 (38.5)	4.2 (0.6)	200,000	582,794	IGL
D9-4	strain	3.0	203.7 (29,545.5)	198.5 (28,782.8)	0.196%	0.057%	0.071%	276.2 (40.1)	4.1 (0.6)	200,000	391,020	IGL
D9-20	strain	3.0	205.6 (29,819.7)	189.4 (27,471.2)	0.197%	0.063%	0.065%	266.0 (38.6)	6.3 (0.9)	200,000	363,644	IGL
D9-9	strain	3.0	212.5 (30,818.2)	205.9 (29,866.8)	0.149%	0.030%	0.030%	238.0 (34.5)	11.0 (1.6)	470,674	10,000,000	None
	load	30						238.0 (34.5)	0.0 (0.0)			
D9-11	load	30						241.9 (35.1)	0.0 (0.0)		2,836,410	IGL
D9-21	load	30						241.9 (35.1)	0.0 (0.0)		10,000,000	None



1000
100
1E+2
1E+3
1E+4
1E+5
1E+6
1E+7
True Stress Amplitude, Δσ/2 (MPa)
Reversals to Failure, 2N_f
Δσ/2 =1099.9 (2N_f)^-0.1077
σ_f ' = 1099.9MPa
b = -0.1077
R^2 = 0.9895
Fatigue Data
Least Squares Fit


10.000%
1.000%
0.100%
0.010%
0.001%
1E+2
1E+3
1E+4
1E+5
1E+6
1E+7
Reversals to Failure, $2N_f$
True Plastic Strain Amplitude, $\Delta\varepsilon_p/2$ (%)
$\Delta\varepsilon_p/2 = 0.6621\ (2N_f)^{-0.5418}$
$\varepsilon_f' = 0.6621$
$c = -0.5418$
$R^2 = 0.9981$
Fatigue Data
Least Squares Fit



10.00%
1.00%
0.10%
0.01%
True Strain Amplitude, Δε/2, %
1E+2
1E+3
1E+4
1E+5
1E+6
1E+7
1E+8
Reversals to Failure, $2N_f$
Δε / 2
$\Delta\varepsilon_p/2$
$\Delta\varepsilon_e/2$
Strain-Life Equation
Fatigue Data (Plastic)
Fatigue Data (Elastic)
Fatigue Data (Total)

- Strain-life fatigue data for selected engineering alloys are included in Table A.2.
- These properties are obtained from small, polished, unnotched axial fatigue specimens under constant amplitude fully reversed cycles of strain.
- Material properties in Table A.2 also omit influences of surface finish, size, stress concentration, temperature, and corrosion.

Table A.2 Monotonic, Cyclic, and Strain-life Properties of Selected Engineering Alloys[a,b,c]

Material	Process Description	S_u MPa (ksi)	HB	E GPa (ksi $\cdot 10^3$)	%RA	S_y/S_y' MPa (ksi)	K/K' MPa (ksi)	n/n'	$\varepsilon_f/\varepsilon_f'$	σ_f/σ_f' MPa (ksi)	b	c
Steel												
1010	HR sheet	331	—	203	80	200/—	534/867	0.185/0.244	1.63/0.104	—/499	-0.100	-0.408
		(48)		(29.5)		(29)/ —	(78)/(126)			—/(72)		
1020	HR sheet	441	109	203	62	262/—	738/1962	0.190/0.321	0.96/0.337	—/1384	-0.156	-0.485
		(64)		(29.5)		(38)/ —	(107)/(284)			—/(201)		
1038[c]	Normalized	582	163	201	54	331/342	1106/1340	0.259/0.220	0.77/0.309	898/1043	-0.107	-0.481
		(84)		(29.5)		(48)/(50)	(160)/(195)			(130)/(151)		
1038[c]	Q & T	649	195	219	67	410/364	1183/1330	0.221/0.208	1.10/0.255	1197/1009	-0.097	-0.460
		(94)		(31.5)		(60)/(53)	(172)/(193)			(174)/(146)		
Man-Ten	HR sheet	510	—	207	64	393/372	—/786	0.20/0.11	1.02/0.86	814/807	-0.071	-0.65
		(74)		(30)		(57)/(54)	—/(114)			(118)/(117)		
RQC-100	HR sheet	931	290	207	64	883/600	1172/1434	0.06/0.14	1.02/0.66	1330/1240	-0.07	-0.69
		(135)		(30)		(128)/(87)	(170)/(208)			(193)/(180)		
1045	Annealed	752	225	—	44	517/—	—/1022	—/0.152	0.58/0.486	—/916	-0.079	-0.520
		(109)		—		(75)/—	—/(148)			—/(133)		
1045	Q & T	1827	500	207	51	1689/—	—/3371	0.047/0.145	0.71/0.196	—/2661	-0.093	-0.643
		(265)		(30)		(245)/ —	—/(489)			—/(386)		
1090[c]	Normalized	1090	259	203	14	735/545	1765/1611	0.158/0.174	0.15/0.250	—/1310	-0.091	-0.496
		(158)		(29.5)		(107)/(79)	(256)/(234)			—/(190)		
1090[c]	Q & T	1147	309	217	22	650/627	1895/1873	0.165/0.176	0.24/0.700	—/1878	-0.120	-0.600
		(166)		(31.5)		(94)/(91)	(275)/(272)			—/(273)		
1141[c]	Normalized	789	229	220	47	493/481	1379/1441	0.187/0.177	0.64/0.602	1117/1326	-0.103	-0.581
		(115)		(32)		(72)/(70)	(200)/(209)			(162)/(192)		
1141[c]	Q & T	925	277	227	59	814/591	1205/1277	0.074/0.124	0.88/0.309	1405/1127	-0.066	-0.514
		(134)		(33)		(118)/(86)	(125)/(185)			(204)/(164)		
4142	Q & T	1413	380	207	48	1378/—	—/2266	0.051/0.124	0.65/0.637	—/2143	-0.094	-0.761
		(205)		(30)		(200)/—	—/(387)			—/(311)		
4142	Q & T	1929	475	207	35	1722/—	—/2399	0.048/0.094	0.43/0.331	—/2161	-0.081	-0.854
		(280)		(30)		(250)/—	—/(348)			—/(314)		
4340	HR	827	243	193	43	634/—	—/1337	—/0.168	0.57/0.522	—/1198	-0.095	-0.563
		(120)		(28)		(92)/—	—/(194)			—/(174)		

4340	Q & T	1240	350	193	57	1178/—	1580/1887	0.066/0.137	0.84/1.122	—/1917	-0.099	-0.720
		(180)		(28)		(171)/—	(229)/(274)			—/(278)		
4340	Q & T	1468	409	200	38	1371/—	—/1996	—/0.135	0.48/0.640	—/1879	-0.086	-0.636
		(213)		(29)		(199)/—	—(290)			—/(273)		
0030	Cast	496	137	207	46	303/320	—/738	—/0.136	0.62/0.280	750/655	-0.083	-0.552
		(72)		(30)		(44)/(46)	—/(107)			(109)/(95)		
8630	Cast	1144	305	207	29	985/682	—/1502	—/0.122	0.35/0.420	1268/1936	-0.121	-0.693
		(166)		(30)		(143)/(99)	—/(218)			(184)/(281)		
304	Annealed	572	—	190	—	276/—	—/2275	—/0.334	—/0.174	—/1267	-0.139	-0.415
		(83)		(27.5)		(40)/—	—/(330)			—/(184)		
304	CD	951	327	172	69	744/—	—/2270	—/0.176	1.16/0.554	—/2047	-0.112	-0.635
		(138)		(25)		(108)/—	—/(329)			—/(297)		
Aluminum												
2024-T3	—	469	—	70	24	379/427	455/655	0.032/0.065	0.28/0.22	558/1100	-0.124	-0.59
		(68)		(10)		(55)/(62)	(66)/(95)			(81)/(160)		
5456-H311	—	400	95	69	35	234/—	—/817	—/0.145	0.42/1.076	—/826	-0.115	-0.797
		(58)		(10)		(34)/—	—/(118)			—/(120)		
7075-T6	—	579	—	70	34	469/524	827/—	0.11/0.146	0.41/0.19	745/1315	-0.126	-0.52
		(84)		(10)		(68)/(76)	(120)/—			(108)/(191)		
A356	Cast	283	93	70	5.7	229/295	388/379	0.083/0.043	0.06/0.027	274/594	-0.124	-0.530
		(41)		(10)		(33)/(43)	(56)/(55)			(40)/(86)		
Others												
AZ91E-T6	Cast Mg.	318	—	45	13	142/180	639/552	0.137/0.184	0.14/0.089	356/831	-0.148	-0.451
		(46)		(6.5)		(21)/(26)	(92)/(80)			(52)/(121)		
Incon 718	Aged	1304	—	204	—	1110/—	—/1986	—/0.112	—/3.637	—/2295	-0.100	-0.894
		(189)		(29.5)		(161)/—	—/(288)			—/(333)		

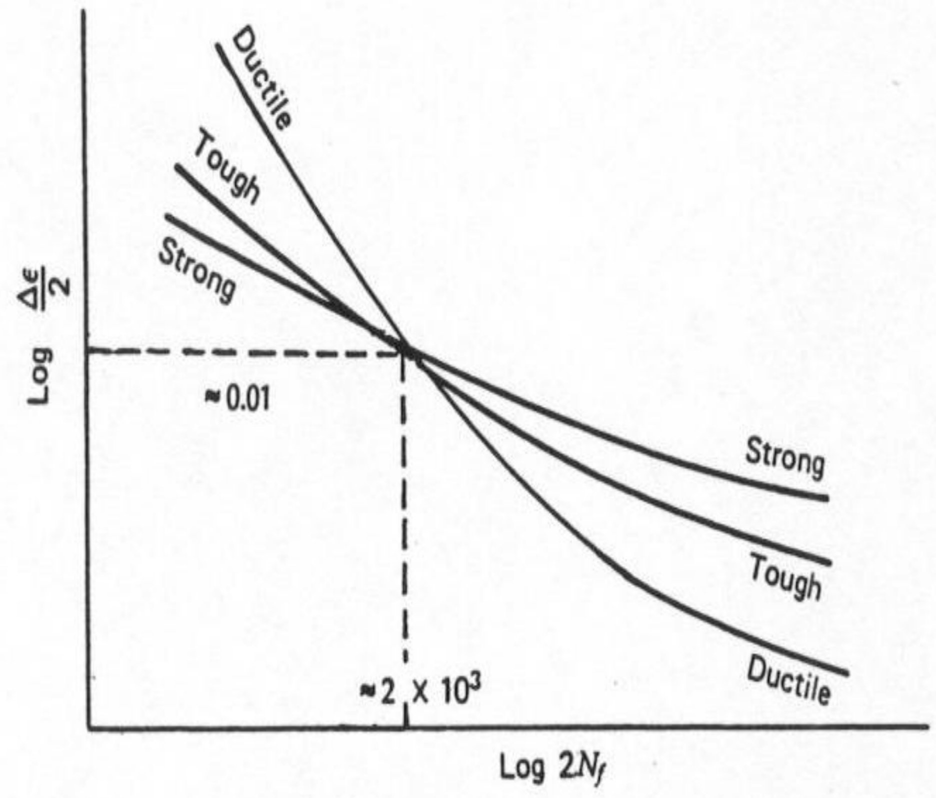
[a] These values do not represent final fatigue design properties. J1099 states "Information presented here can be used in preliminary design estimates of fatigue life, the selection of materials and the analysis of service load and/or strain data."

[b] "Technical Report on Low Cycle Fatigue Properties, Ferrous and Non-Ferrous Materials," SAE J1099, 1998 and 1975. (With permission of the Society of Automotive Engineers).

[c] M.L. Roessle, Correlations Among Microstructural Parameters, Trensile Data, And Fatigue Properties For Steel, MS Thesis, University of Toledo, 1998.

ε -*N* APPROACH TO LIFE ESTIMATION

- Many materials have similar life at a total strain amplitude of about 0.01.
- At larger strains, increased life is dependent more on ductility, while at smaller strains longer life is obtained from higher strength materials.
- The optimum overall strain-life behavior is for **tough metals**, which are materials with good combinations of strength and ductility.



ε -*N* APPROACH TO LIFE ESTIMATION

- The strain-based approach is a comprehensive approach which can be applied to both low cycle and high cycle fatigue regimes.

- For long life applications where the plastic strain term is negligible, the total strain-life equation reduces to Basquin's Eq. which was also used for the stress-life (*S-N*) approach.

$$\frac{\Delta\varepsilon}{2} = \varepsilon_a = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = \frac{\sigma'_f}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c$$

$$\frac{\Delta\sigma}{2} = \sigma_a = \sigma'_f\left(2N_f\right)^b$$

DETERMINATION OF STRAIN-LIFE FATIGUE **PROPERTIES**

- The **fatigue strength coefficient, σ_f'**, and **fatigue strength exponent, *b***, are the intercept and slope of the linear least squares fit to stress amplitude, $\Delta\sigma/2$, versus reversals to failure, $2N_f$, using a log-log scale.

- Similarly, the **fatigue ductility coefficient, ε_f'**, and **fatigue ductility exponent, *c***, are the intercept and slope of the linear least squares fit to plastic strain amplitude, $\Delta\varepsilon_p/2$, versus reversals to failure, $2N_f$, using a log-log scale.

- Plastic strain amplitudes can either be measured directly from half the width of stable hysteresis loops, or calculated from

$$\frac{\Delta\varepsilon_p}{2} = \frac{\Delta\varepsilon}{2} - \frac{\Delta\sigma}{2E}$$

DETERMINATION OF STRAIN-LIFE FATIGUE **PROPERTIES**

- When fitting the data to obtain the four strain-life properties, stress or plastic strain amplitude should be treated as independent variables, whereas the fatigue life is the dependent variable (i.e. fatigue life cannot be controlled and is dependent upon the applied strain amplitude).

- The cyclic strength coefficient, K′, and cyclic strain hardening exponent, n′, are obtained from fitting stable stress amplitude versus plastic strain amplitude data. Rough estimates of K′ and n′ can also be calculated from the low cycle fatigue properties by using:

$$K' = \frac{\sigma'_f}{(\varepsilon'_f)^{\frac{b}{c}}} \qquad n' = \frac{b}{c}$$

These equations are derived from compatibility between strain-life equations.

ESTIMATION OF STRAIN-LIFE FATIGUE PROPERTIES

- Strain-life equation has been approximated from monotonic tensile properties (**method of Universal Slopes**):

$$\frac{\Delta\varepsilon}{2} = 0.623\left(\frac{S_u}{E}\right)^{0.832}\left(2N_f\right)^{-0.09} + 0.0196\left(\varepsilon_f\right)^{0.155}\left(\frac{S_u}{E}\right)^{-0.53}\left(2N_f\right)^{-0.56}$$

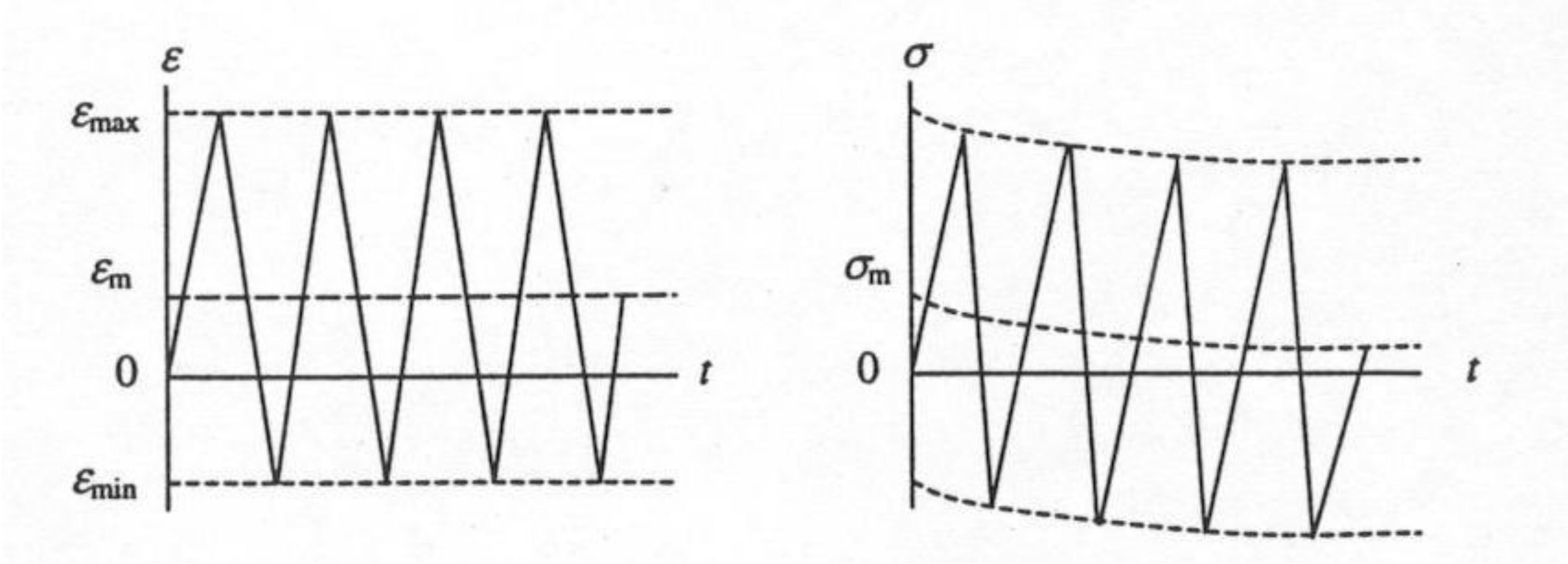
- Another approximation that only uses hardness and *E* has been shown to provide good agreement with experimental data for steels and is given by (**Roessle and Fatemi**, 2000):

$$\frac{\Delta\varepsilon}{2} = \frac{4.25\left(HB\right)+225}{E}\left(2N_f\right)^{-0.09} + \frac{0.32\left(HB\right)^2 - 487\left(HB\right)+191000}{E}\left(2N_f\right)^{-0.56}$$

MEAN STRESS EFFECTS

- Strain-controlled cycling with a mean strain usually results in a mean stress which may fully or partially relax with continued cycling.

- The relaxation is due to the presence of plastic deformation, and therefore, the rate or amount of relaxation depends on the magnitude of the plastic strain amplitude (more mean stress relaxation at larger strain amplitudes).



MEAN STRESS EFFECTS

- Stress relaxation is different from the cyclic softening phenomenon and can occur in a cyclically stable material.

- Mean strain does not usually affect the fatigue behavior unless it results in a non-fully relaxed mean stress.

- Since there is more mean stress relaxation at higher strain amplitudes due to larger plastic strains, mean stress effect on fatigue life is smaller in the low cycle fatigue region and larger in the high cycle fatigue region (Fig. 5.15).

MEAN STRESS EFFECTS

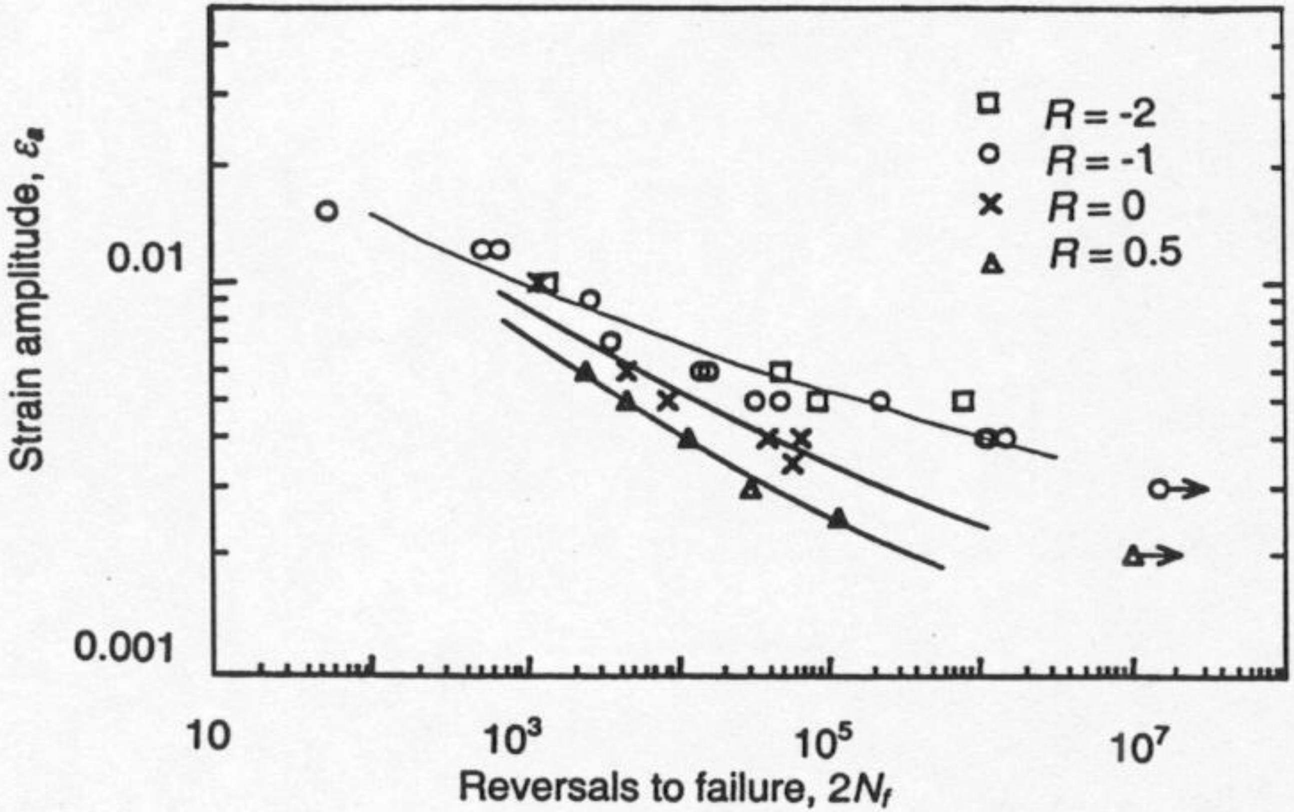


Figure 5.15 Effect of mean strain on fatigue life for SAE 1045 hardened steel [23] (reprinted with permission of Elsevier Science).

MEAN STRESS EFFECTS

- Several models dealing with mean stress effects on strain-life fatigue behavior are available.

  - **Morrow's parameter**:

$$\frac{\Delta\varepsilon}{2} = \varepsilon_a = \frac{\sigma'_f - \sigma_m}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c$$

σ_m is the mean stress.

  - An alternative version of **Morrow's mean stress parameter** where both the elastic and plastic terms are affected by the mean stress is given by:

$$\frac{\Delta\varepsilon}{2} = \varepsilon_a = \frac{\sigma'_f - \sigma_m}{E}\left(2N_f\right)^b + \varepsilon'_f\left(\frac{\sigma'_f - \sigma_m}{\sigma'_f}\right)^{\frac{c}{b}}\left(2N_f\right)^c$$

MEAN STRESS EFFECTS

- Another equation suggested by **Smith, Watson, and Topper** (often called the SWT parameter) is:

$$\sigma_{\max}\,\varepsilon_a E = \left(\sigma'_f\right)^2 \left(2N_f\right)^{2b} + \sigma'_f \varepsilon'_f E \left(2N_f\right)^{b+c}$$

where σ_{max} = $\boldsymbol{\sigma_m}$ + σ_a and ε_a is the alternating strain.

  - This equation is based on the assumption that for different combinations of strain amplitude, ε_a, and mean stress, σ_m, the product $\sigma_{max}\,\varepsilon_a$ remains constant for a given life.
  - If σ_{max} is zero, this Eq. predicts infinite life, which implies that tension must be present for fatigue fractures to occur.
  - The SWT Eq. has been shown to correlate mean stress data better for a wide range of materials and is regarded to be more promising for general use.

FACTORS INFLUENCING ε -*N* BEHAVIOR

- Similar to the *S-N* approach, in addition to the mean stress, many other factors can influence strain-life fatigue behavior of a material. These include:
  - stress concentrations (Ch. 7),
  - residual stresses (Ch 8),
  - multiaxial stress states (Ch 10),
  - environmental effects (Ch 11),
  - size effects (similar to those in the *S-N* approach), and
  - surface finish effects.
- The effects of many of these factors are similar to those on the *S-N* behavior.

SURFACE FINISH EFFECT ON ε -N BEHAVIOR

- Due to large plastic strains in the low cycle region, there is usually little influence of surface finish at short lives.

- There is more influence of surface finish in the high cycle fatigue regime where elastic strain is dominant.

- Therefore, only the elastic portion of the strain-life curve is modified to account for the surface finish effect.

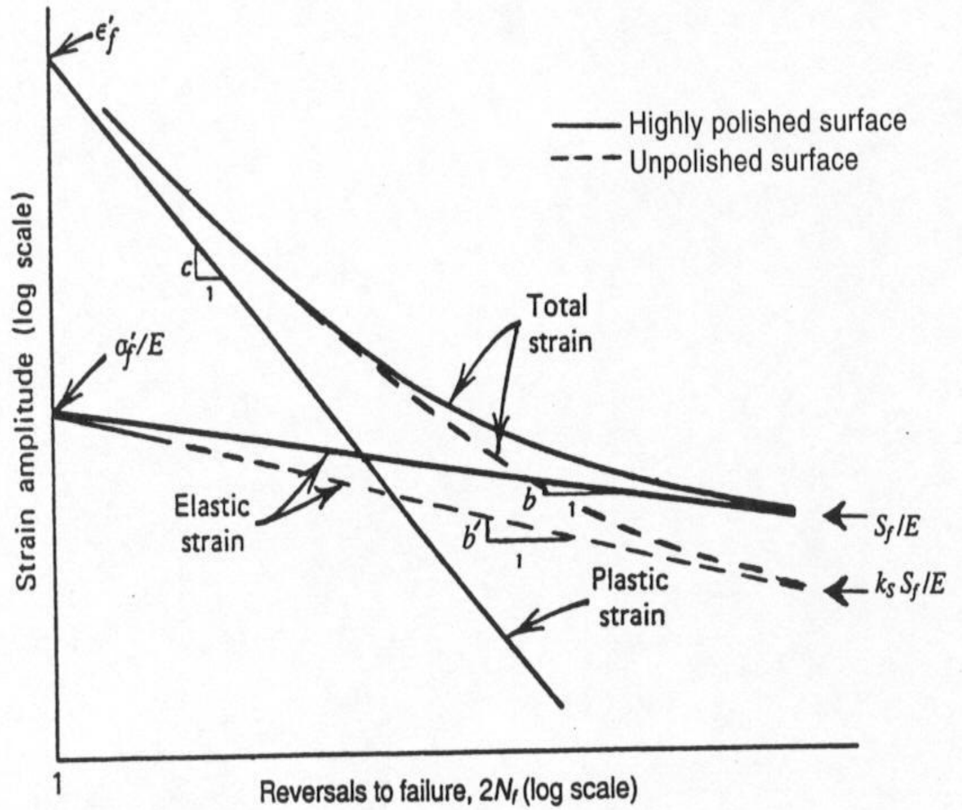
SURFACE FINISH EFFECT ON ε -*N* BEHAVIOR

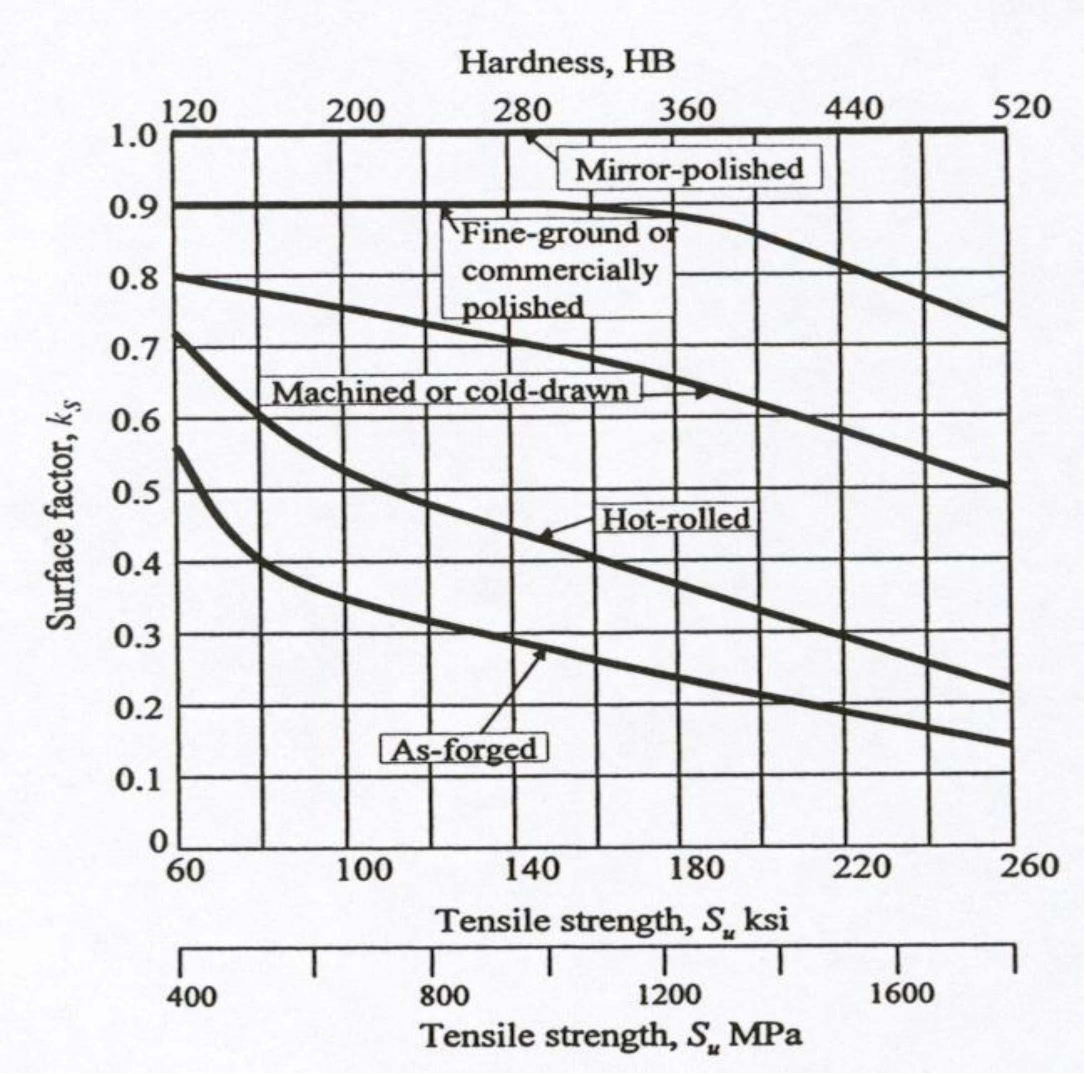
- This is done by reducing the slope of the elastic strain-life curve, *b*, analogous to the modification of the *S-N* curve for surface finish.

- Surface finish correction factors for steels are given in Fig. 4.15.

- The slope b' for steels with fatigue limit assumed at 10^6 cycles can be calculated from $b' = b + 0.159 \log k_s$.

- This correction can also be used for nonzero mean stress loadings.




Hardness, HB
120
200
280
360
440
520
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Mirror-polished
Fine-ground or
commercially
polished
Machined or cold-drawn
Hot-rolled
As-forged
Surface factor, k_S
60
100
140
180
220
260
Tensile strength, S_u ksi
400
800
1200
1600
Tensile strength, S_u MPa

SUMMARY

- Basic material mechanical properties such as strength and ductility can be obtained from simple monotonic tensile tests. However, The stress-strain behavior obtained from simple monotonic tensile tests can be quite different from that obtained under cyclic loading.

- Cyclic loading can cause hardening and/or softening of the material. Using a monotonic stress-strain curve of a cyclic softening material in a cyclic loading application can significantly underestimate the extent of plastic deformation present.

- Changes in cyclic deformation behavior are more pronounced at the beginning of cyclic loading, as the material usually gradually stabilizes with continued cycling.

SUMMARY

- The stable cyclic stress-strain curve can be represented by the following equation for many metals:

$$\varepsilon_a = \frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{K'}\right)^{1/n'}$$

  where K′ and n′ are material cyclic deformation properties.

- Fatigue cracks usually nucleate from plastic straining in localized regions. Strain-based approach to fatigue problems is widely used at present.

SUMMARY

- The strain-life equation is expressed as:

$$\frac{\Delta\varepsilon}{2} = \varepsilon_a = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = \frac{\sigma'_f}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c$$

- The Strain-life approach is a comprehensive approach which can be applied for the treatment of both low cycle and high cycle fatigue.

- In the low cycle region plastic strain is dominant, whereas in the high cycle region elastic strain is dominant.

SUMMARY

- At large strains better fatigue resistance depends more on ductility, while at small strains it depends more on strength.
- Strain-controlled cycling with a mean strain results in a mean stress that usually relaxes at large strain amplitudes due to the presence of plastic deformation.
- A non-relaxing mean stress can significantly affect the fatigue life with tensile mean stress having detrimental effect and compressive mean stress having beneficial effect.
- Other synergistic effects of loading, environment, and component or material processing can also influence the strain-life behavior.

EXAMPLE PRBLEM USING STRAIN-LIFE APPROCH